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Optimization of Sensor Placement for Health Monitoring of Static Plane Truss Structures

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Abstract

Structural health monitoring of buildings is critical due to aging infrastructure and natural disasters. The arrangement of sensors is crucial for quality monitoring. Despite the limited number of sensors, their placement should reflect structural characteristics to determine health conditions. This study simplifies the problem by employing the MKE and EFI methods for static planar truss structures based on internal forces. The two methods were evaluated by finite element analysis of the structure using the Modal Confidence Criterion (MAC), and the comparison revealed that the sensor placement obtained from the EFI is a more accurate reflection of structural damage.

Keywords

Structural Health Monitoring; Sensor Placement; Static Planar Truss; MKE; EFI; MAC.

1. Introduction

Presently, the infrastructure construction in some areas is relatively complete, yet concurrently, a certain number of buildings are in a state of disrepair. Furthermore, in recent years, natural disasters such as earthquakes and floods have caused varying degrees of damage to some building structures. Consequently, monitoring and diagnosing the performance of building structures, timely detection of structural damage, and predicting and evaluating the safety of possible disasters have become an indispensable requirement for future projects. Additionally, this field represents a crucial area of development within the discipline of civil engineering in the future.

Structural health monitoring aims to detect, locate, and quantify any potential damage, thereby ensuring the safety and longevity of the structure [1]. Theoretically, incorporating multiple sensors in a structural health monitoring system enhances the accuracy of structural characterization, as it allows for comprehensive coverage and better identification of anomalies. However, practical constraints, such as cost, accessibility, and physical limitations, restrict the number of sensors that can be deployed in real-world applications. Therefore, sensor placement must be carefully planned to ensure effective monitoring, leading to the optimization challenge of sensor placement. This optimization involves determining the most strategic locations for sensors to maximize the quality of data collected while minimizing redundancy and costs.

The optimal arrangement of sensors in large flexible structures (mainly space structures in aviation and aerospace) has been widely studied, and numerous computational methods for optimizing measurement points have been proposed. These include the modal kinetic energy method (MKE), the effective independence method (EFI), the Guyan reduction method, and the genetic algorithm (GA) [2-5]. Despite significant advancements in research, practical issues such as high implementation costs, limited access, and environmental constraints pose considerable challenges to optimal sensor placement in real-world applications. These challenges and the need for precise damage detection form the core motivation behind this study.

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In this study, the theoretical formulas of the MKE and the EFI are derived, and a planar truss structure is used as a case study to illustrate the optimal arrangement of sensors by applying the aforementioned two methods. The differences between the two optimization algorithms are compared, and the optimization effect is evaluated using the modal assurance criterion. Moreover, the static planar truss serves as the basis for studying complex models, which is significant for future health monitoring of large and complex truss structure buildings.

2. Optimal Sensor Placement Theory

2.1 The EFI Method

The core concept of the EFI method is to commence with all potential measurement points and remove the degrees of freedom that result in the least change in the determinant value of the Fisher information matrix subsequently, while retaining the measurement points with the greatest contribution of the target modes to the linear irrelevance, with the aim of achieving the most accurate estimation of the modal shapes. In other words, the method aims to retain as much as possible the linear irrelevance information, given the constraints of the sensor placement.

Define the sensor output response as U_s, then:

$$U_s = \Phi_s q = \sum_{i=1}^{N} \Phi_i q_i \tag{1}$$

In this formula, the variable q represents the modal coordinate, Φ_s denotes the measured n×N order modal matrix, Φ_i signifies the i-th order modal shape, and q_i is the mode participation coefficient.

The least squares solution of Eq. (1) for the modal coordinates q is given by:

$$\hat{q} = [\Phi_s^T \Phi_s]^{-1} \Phi_s^T U_S \tag{2}$$

Considering the noise S, the response can be expressed as:

$$U_s = \Phi_s q + S = \sum_{i=1}^{N} \Phi_i q_i + S \tag{3}$$

For the bias present in $\hat{q}$, assuming unbiased efficient estimation, the covariance matrix P of the estimated bias can be expressed as:

$$P = E[(q - \hat{q})(q - \hat{q})^T] = Q^{-1} \tag{4}$$

$$Q = \frac{1}{\sigma^2} \Phi_s^T \Phi_s = \frac{1}{\sigma^2} A_0 \tag{5}$$

Where Q is the Fisher information matrix. Q is maximized when A_0 is maximized, so the matrix E can be constructed by reflecting Q in terms of A_0.

$$(A_0 - \lambda I)\psi = 0 \tag{6}$$

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Where λ and ψ are the eigenvalues and eigenvectors of the matrix A_0 separately. It can be deduced that:

$$\psi^T \lambda^{-1} \psi = A_0^{-1} \tag{7}$$

Construct the matrix E:

$$E = \Phi_s \psi \lambda^{-1} (\Phi_s \psi)^T = \Phi_s A_0^{-1} \Phi_s^T = \Phi_s [\Phi_s^T \Phi_s]^{-1} \Phi_s^T \tag{8}$$

E is an idempotent matrix where the i-th element on its diagonal represents the contribution of the i-th degree of freedom or test point to the rank of matrix Φ_s, and ultimately to matrix A_0. Thus, E characterizes the effective independent distribution of the candidate sensor positions, with the diagonal elements indicating the linearly independent contributions of each sensor candidate measurement point to the modal matrix. Once matrix E is obtained through Eq. (6), each candidate point is prioritized based on the value of its diagonal element. An iterative calculation is then implemented in MATLAB, where the candidate point corresponding to the smallest diagonal element is removed in each iteration until the desired number of points is reached. This process helps to ensure an optimal sensor distribution that provides meaningful information while balancing system complexity and performance.

To better understand the EFI method, the following pseudocode for the sensor placement process is provided:

Algorithm 1 Optimal Sensor Placement using EFI Method

1: **Construct Modal Matrix Φ**

2: Concatenate the displacement matrices in the x-direction (Ux) and y-direction (Uy) to form the modal matrix Φ.

3: Φ is a matrix of size 2n × N , where n is the number of nodes and N is the number of modes.

4: **Initialize Candidate Sensor Set**

5: Compute the Fisher Information Matrix Q = ΦT Φ.

6: Initialize the candidate sensor set, which includes all sensor positions.

7: **Iteratively Remove Sensors Until Desired Number is Achieved**

8: while number of sensors > desired number do

9: Compute the matrix E = Φ · Q−1 · ΦT .

10: Identify the sensor position corresponding to the smallest diagonal element in matrix E.

11: Remove the sensor with the smallest contribution from the candidate set.

12: Remove the corresponding row from the modal matrix Φ.

13: Update the Fisher Information Matrix Q = ΦT Φ.

14: **end while**

15: **Terminate Iteration**

16: Output the indices of the remaining selected sensors.

2.2 The MKE Method

The basic concept is that the response in the degree of freedom with the larger modal strain energy is also larger. Consequently, mapping the respective modal strain energy distribution for each target

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mode of vibration and then placing the sensors at these positions will enable the identification of the relevant parameters.

The procedure of MKE for sensor placement is similar to EFI, with the main distinction being that the aim of MKE is to maximize modal strain energy, whereas the purpose of EFI is to maximize the linear irrelevance of target modal vectors. Define the MKE as:

$$KE = \Phi^T M \Phi \tag{9}$$

Applying the Cholesky decomposition to the mass matrix M,

$$M = LL^T \tag{10}$$

Then:

$$KE = \Phi^T LL^T \Phi = (L\Phi)^T L\Phi \tag{11}$$

Let $\psi = L\Phi$,

$$KE = \psi^T \psi \tag{12}$$

As can be observed from Eq. (12), the formula for the modal strain energy is similar to the Fisher information matrix. Consequently, the process of iterations can be performed in a similar way to the EFI method, whereby the degrees of freedom with the lowest modal energy are gradually removed until the sensor design requirements are met.

3. Modal Assurance Criterion

The modal assurance criterion (MAC) is an effective method for assessing the correlation between two vectors [6], which can be expressed using the following formula:

$$MAC_{ij} = \frac{(\Phi_i^T \Phi_j)^2}{(\Phi_i^T \Phi_i)(\Phi_j^T \Phi_j)} \tag{13}$$

Φ_i and Φ_j represent the i-th and j-th order modal vectors, respectively. The Modal Assurance Criterion (MAC_{ij}) ranges from 0 to 1, where a value of 0 indicates that the two vectors are orthogonal, and a value of 1 signifies complete correlation. Thus, larger non-diagonal MAC values imply higher correlation between vectors and a reduced ability to represent the unique characteristics of the original structure. Conversely, smaller non-diagonal MAC values indicate lower correlation and a better ability to capture the distinct features of the structure [7]. In general, if the MAC value exceeds 0.9, the vectors are considered highly correlated and indistinguishable, whereas values below 0.25 suggest that the vectors are approximately orthogonal [8].

4. Example and Evaluation of Effectiveness

This paper employs a simply supported I-beam steel truss bridge structure as an example, optimizing the sensor placement through the application of EFI and MKE. The truss model was constructed using the finite element analysis software ABAQUS. The cross-section of each member of the truss is

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identical, with a cross-sectional area of 0.01 m², a length of 100 m, a density of 7800 kg/m³, a modulus of elasticity of 210 GPa, a Poisson's ratio of 0.3, and a planar truss dimension as illustrated in Fig. 1. Each node of a planar truss has degrees of freedom in 2 directions, e.g., the degrees of freedom of 1 node are expressed as 1x and 1y, and so on.

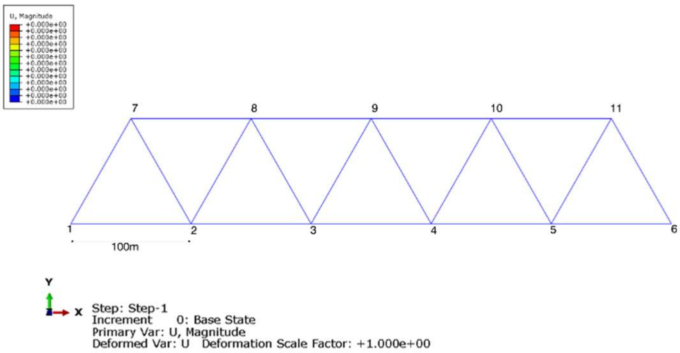

Figure 1. Drawing of finite element model

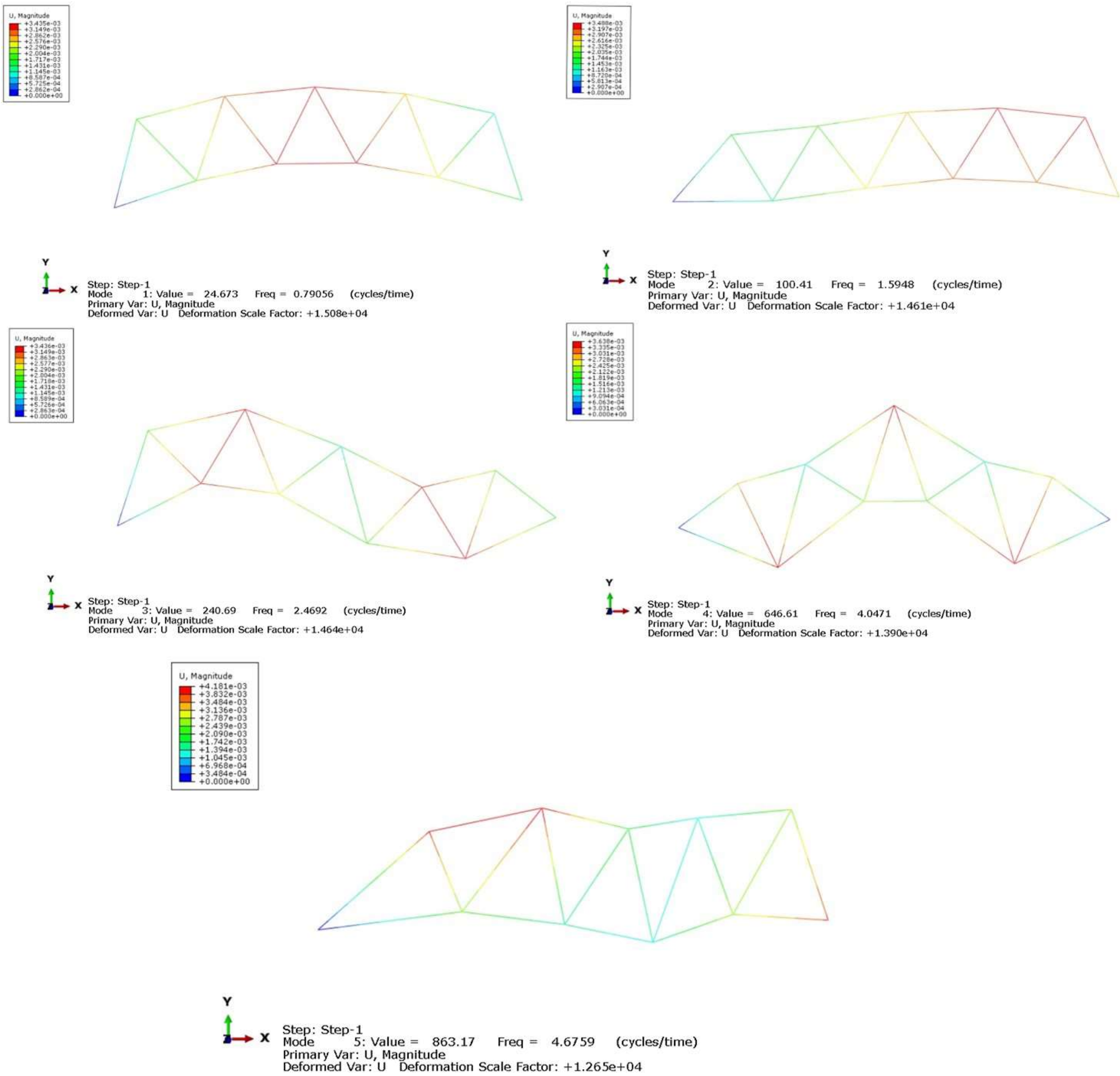

Figure 2. Modal shapes and frequencies of the first five orders of the truss structure

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To improve transparency and repeatability, additional details regarding the finite element analysis are provided. The truss structure was modeled in ABAQUS with specific boundary conditions: the left end of the truss was fixed in both x and y directions, while the right end was allowed to move along the x-axis but was fixed along the y-axis.

4.1 Comparison of Two Methods of Placement

The example structure was optimized with 12, 10 and 8 sensors, respectively placed in EFI and MKE, and the optimization results are presented in Table 1-3.

Table 1. Comparison of optimal results of arranging twelve sensors

	Sensor placement results (12 sensors)											
EFI	5x	6x	7x	8x	11x	2y	3y	4y	5y	8y	9y	10y
MKE	5x	6x	7x	11x	2y	3y	4y	5y	7y	9y	10y	11y

Table 2. Comparison of optimal results of arranging ten sensors

	Sensor placement results (10 sensors)											
EFI		6x	7x	8x	11x	2y	4y	5y	8y	9y	10y	
MKE		5x	6x	7x	11x	2y	3y	4y	5y	10y	11y	

Table 3. Comparison of optimal results of arranging eight sensors

	Sensor placement results (8 sensors)											
EFI			6x	7x	8x	11x	2y	5y	8y	9y		
MKE			6x	7x	11x	2y	3y	4y	5y	10y		

4.2 Assessment of Optimizations Effectiveness

The optimization results of the two methods for the deployment of 8 sensors are evaluated using the modal assurance criterion. The corresponding modal vectors are extracted from the sensor placement results of the above two optimization methods by the MATLAB program and the values of the MAC between each order are calculated and finally the mean square error (MSE) of the values of the MAC is obtained. The MSE is compared to getting the better optimization method reflecting the condition of the structure. Results can be obtained and are shown in Table 4.

Table 4. Comparison of mean square error

Optimization	MAC Value				MSE
	Mode 1-2	Mode 2-3	Mode 3-4	Mode 4-5	
EFI	0.0083	0.0028	0.0167	0.0001	0.0064
MKE	0.0434	0.0254	0.0066	0.0003	0.0169

The results in Table 4 show that the mean square error of EFI is 0.0064, which is smaller than that of MKE (0.0169).Therefore, the sensor arrangement effect of EFI is better than that of MKE in a simple static planar truss.

It is necessary to further discuss why the EFI method outperformed the MKE method in this example. The EFI method maximizes the independent contribution of each sensor to the structure, which is

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highly effective in cases with significant modal redundancy. On the other hand, the MKE method focuses on maximizing modal strain energy, which may not be as effective at distinguishing closely spaced modes. This makes EFI more suitable for capturing independent modal information.

5. Conclusion

The EFI and KEM methods are two approaches to optimal sensor placement. The former employs a gradual removal of degrees of freedom that minimizes the change in the determinant value of the Fisher information matrix, resulting in optimal placement. In contrast, the latter method retains measurement points with a larger energy distribution from an energy perspective, thereby achieving optimal placement. The results demonstrate that the optimization outcomes obtained by the EFI method more accurately reflect the characteristics of the original structure.

Future work could explore the integration of machine learning techniques, such as reinforcement learning, to further enhance sensor placement optimization. Additionally, expanding the current methodology to complex three-dimensional truss structures and incorporating environmental factors like temperature variations and loading conditions would significantly improve the practical application of sensor optimization in real-world scenarios. These expansions would create a more comprehensive framework to address the challenges of structural health monitoring.

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