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Based on Event-Triggered Sliding Mode Control Second-order of Multi-UAV Leader-Follower Consensus

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Abstract

The study investigates the leader-following consensus problem of second-order multi-unmanned aerial vehicle (UAV) systems under fixed directed topology. To address the system's disturbances and uncertainties, a sliding mode control method is considered to ensure robustness against external perturbations. To conserve network and computational resources, an event-triggered sliding mode control algorithm for leader-following consensus is proposed. For each follower UAV, a trigger function based on the state error is designed, which takes a value of zero only when the state error meets specific conditions, thereby triggering events and updating and transmitting their own sampled information. Between two adjacent event-triggering moments, the control input is only influenced by the leader's control signal. The stability of the multi-UAV controller is verified using Lyapunov stability theory, proving the system's stability. Subsequently, a simulation of a formation of six UAVs was conducted to validate the practicality of the theoretical results.

Keywords

Leader-following; Consensus of Second-order Multi-UAV Systems; Event-Triggered Control; Distributed Sliding-Mode Control.

1. Introduction

Recent years have seen rapid development in multi-agent systems, particularly in the field of cooperative control, where cooperative control replaces individual entities to address large and complex real-world problems [1-3]. Thanks to the swift advancement in computer science and network communication technology, drones have become a quintessential example in multi-agent systems [4-6]. These drones are not only capable of working together to respond to search and rescue missions in emergency situations but also monitoring extensive environmental changes.

This paper extends the multi-agent theory to unmanned aerial vehicle (UAV) formation systems. Within the realm of multi-UAV collaboration, the leader-follower control strategy is a prevalent and effective method of coordination[55555].The central premise of this approach is that within a fleet of UAVs, one or several are designated as leaders, while the rest assume the role of followers, executing the commands and task assignments of the leaders. This hierarchical structure enhances the operational efficiency and agility of the entire UAV swarm. In reference [9],the study proposes an adaptive hybrid controller for managing the leader-follower formation of multiple UAVs in the presence of communication delays.This paper introduces a real-time guided point calculation method for leader-follower formation flight control, enabling rapid convergence and precise formation

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maintenance for fixed-wing UAVs. The efficiency and accuracy of this method in various formation transitions are demonstrated through experimental validation in [10].

In practical applications, drone swarms are often confronted with intricate and dynamic scenarios. Nonetheless, sliding mode control has demonstrated a high degree of robustness in coping with uncertainties and disturbances from the external environment.[11-13] This study addresses the issue of time-varying stratigraphic tracking in multi-UAV systems with dynamic inputs. Utilizing the principles of sliding mode control, the research develops protocols for both continuous and discrete-time distributed sliding mode formation control in [14]. In reference [15],a sliding mode control-based Leader-Follower formation controller for UAVs is successfully designed, simulated in MATLAB, and experimentally proven on a three-quadrotor platform, demonstrating its effectiveness and feasibility.

On the other hand, in practical multi-UAV systems, information is exchanged between drones via a communication network. Consequently, the limited channel bandwidth exerts considerable communication pressure on the information transmission between UAVs. Therefore, the adoption of an event-triggered control strategy effectively reduces the demand for communication resources in such scenarios.{16-18}

The paper introduces an event-triggered controller for time-delayed discrete MAS formation, proven stable and communication-reducing in simulations and real UAV tests in [19].This paper presents an event-triggered finite-time privacy-preserving formation control scheme for multi-UAV systems, addressing privacy, bandwidth constraints, and convergence, demonstrating efficacy through simulations and reducing communication overhead in [20]. The article introduces an observer-based event-triggered sliding mode control for secure and efficient multi-UAV formation tracking resilient to replay attacks, optimizing communication and ensuring performance in [21]..Reference[22] examines the consensus tracking issue for second-order multiagent systems in the context of channel fading. To ease network congestion, an event-triggered communication strategy is implemented to lower the frequency of information exchange between agents, and a sliding mode controller with event triggering is subsequently developed.

This paper investigates the leader-follower consensus problem in second-order multi-UAV systems. Distinguishing itself from previous literature, it introduces an innovative integral sliding mode surface that rapidly reduces error. Considering the significant communication pressure among UAVs, a sliding-mode-based distributed event-triggered control is proposed. Utilizing Lyapunov's stability theorem and considering the second-order system model, the feasibility of the controller is demonstrated.

2. Preliminaries and Problem Formulation

2.1 Graph Theory

Consider a multi-UAV system in which N followers are labeled as 1, 2, ..., N, and one leader is designated as 0. The network of interactions among the N followers can be represented by a weighted directed graph $G=\{V,E,A\}$, where $V=\{1,2,\cdots,n,\ 0\}$ represents a set of multi-UAV and $E\subseteq V^2$ is edge set. If UAV i and UAV j can communicate with each other, the symmetric adjacency matrix $A=\left[a_{ij}\right]\in R^{N\times N}$ is defined by a_{ij} > 0 ; otherwise a_{ij} = 0 .For the follower node i , fine its neighbor node set as $\mathcal{N}_i=\left\{j\left(i,\ j\right)\in A\right\}$, A directed path in the graph G is i from node to node j an ordered sequence, namely $\left(i,\ i_1\right),\left(i_1,\ i_2\right),\ldots\left(i_m,\ j\right)\in A$ one $i,\ j,\ i_p\in V$, $p=1,2,\cdots,\ m$ If a node in the graph has a directed path to all other nodes, the node is said to be globally reachable. A graph that contains a directed spanning tree is globally reachable if and only if the graph has at least one root node.

2.2 Multi-UAV System Modeling

In a second-order multi-UAV system with N followers and 1 leader, the dynamic equation for follower i is described as:

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t), \quad i \in \{1,2,3,\ldots,n\} \end{cases} \tag{1}$$

Where $p_i(t) \in R^m$ and $v_i(t) \in R^m$ represent the position and velocity of UAV i at time, respectively. $u_i(t) \in R^m$ serves as the control input protocol for UAV i.

For leader 0, the dynamic equation is described as

$$\begin{cases} \dot{p}_0(t) = v_0(t) \\ \dot{v}_0(t) = u_0(t), \end{cases} \tag{2}$$

where $p_0(t) \in R^m$ and $v_0(t) \in R^m$ represent the position and velocity of leader UAV, respectively. The control input $u_0(t)$ of the leader varies dynamically but is unknown to the followers.

Definition 1.

For each UAV, If there is a control protocol $u_0(t)$ such that the system is satisfied in any initial state

$$\begin{aligned} &\lim_{t\to\infty} \|p_i(t) - p_0(t)\| = 0, \quad i = 1,2,3,\ldots,n \\ &\lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0, \quad i = 1,2,3,\ldots,n \end{aligned} \tag{3}$$

It is said that systems (1) and (2) solve the problem of leader following consistency.

Assumption 1.

The input dynamics of the leader are bounded, denoted as $u_0(t)$, and there exist two constants $u_{min}(t)$ and $u_{max}(t)$, satisfying the condition for all $t \geq 0$, $u_{min}(t) \leq u_0(t) \leq u_{max}(t)$.

Remark 1

Based on the operational dynamics of the sliding mode control system, the control input $u_i(t)$ is divided into two distinct phases: one part is the switching control for the reaching motion phase, and the other part is the equivalent control $u_{i2}(t)$ on the sliding surface for $u_{i1}(t)$.

2.3 Sliding-Model

Sliding Mode Control (SMC) is a dynamic control strategy that forces a system to follow a predefined trajectory by designing a changing structure. The advantages of this method include fast response, insensitivity to parameter variations and disturbances, no need for online system identification, and simple physical implementation. In short, SMC can keep the system stable and respond quickly in the face of uncertainties and interference.

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First, establish the appropriate sliding surfaces. For $t \in \left[t_{k_i}^i, t_{k_{i+1}}^i\right]$, each UAV, the following design is made for the sliding surfaces:

$$\begin{aligned} s_i(t) &= \sum_{j \in \mathcal{N}_i} a_{ij}\left[\left(v_j(t)-v_i(t)\right)+a_{i0}\left(v_i(t)-v_0(t)\right)\right] \\ &\quad -\int_0^t\left(\sum_{j \in \mathcal{N}_i} a_{ij}\left(u_{j2}(\tau)-u_i(\tau)\right)+a_{i0}u_{i2}(\tau)\right)d\tau \end{aligned} \tag{4}$$

where the the equivalent control $u_{i1}(t)$ on the sliding surface isdefined.

Rewrite the compact form of the sliding surface

$$S(t)=(L+B)\otimes I_n\cdot\left(V(t)-1_N\otimes v_0(t)\right)-\int_0^t (L+B)\otimes I_n\cdot U_2(\tau)d\tau \tag{5}$$

where $S(t)=\left[s_1(t)^T, s_2(t)^T, \ldots, s_N(t)^T\right]^T$, $V(t)=\left[v_1(t)^T, v_2(t)^T, \ldots, v_N(t)^T\right]^T$ and $U_2(t)=\left[u_{12}(t)^T, u_{22}(t)^T, \ldots, u_{N2}(t)^T\right]^T$, If the state of Multi-UAV remains on the sliding surfaces , we have $S=\dot{S}=0$, $\dot{V}(t)-1_N\otimes\dot{v}(t)=U_2(t)$.

3. Main Results

3.1 Control Protocol

This section will delve into event-triggered distributed sliding mode control. In the distributed event-triggered strategy, let $\left\{t_k^i\right\}_{k=0}^{\infty}$ denote the triggering time of the event-triggered rule. The control protocol changes at time $t=t_k^i$ and remains constant through zero-order hold (ZOH) until the next triggering instant $t=t_{k+1}^i$. Then, the distributed event-triggered sliding mode controller is designed as follows:

$$u_i(t)=u_{i1}(t)+u_{i2}(t), t\in\left[t_k^i, t_{k+1}^i\right] \tag{6}$$

the equivalent control on the sliding surface $u_{i1}(t)$ isdesigned as

$$u_{i1}(t)=-\sigma sgn(s_i(t))+u_{\min} \tag{7}$$

where σ is a control parameter

Then,the distributed linear consensus controller $u_{i2}(t)$ for the reaching motion stage is designed as

ISSN: 2414-1895 DOI: 10.6919/ICJE.202503_11(3).0002

$$\begin{aligned} u_{i2}(t) &= K_1 \sum_{j \in \mathcal{N}_i} a_{ij} \left[p_j(t_k^i) - p_i(t_k^i) + v_j(t_k^i) - v_i(t_k^i) \right] \\ &\quad - K_2 a_{i0} \left[p_i(t_k^i) - p_0(t) + v_i(t_k^i) - v_0(t) \right] \end{aligned} \tag{8}$$

Theorem 1

Under protocol(6),If the control parameters meet the condition where σ is greater than zero, then within a multi-UAV system, the trajectories of the second-order states, denoted as (1) and (2), will converge to the sliding surface within a finite time period.

Proof. From equation (4), we can conclude that

$$\dot{S}(t) = (L+B) \otimes I_n \left(\dot{V}(t) - 1_N \otimes \dot{v}_0(t) - U_0(t) \right) \tag{9}$$

Where $\dot{V}$ and $\dot{v}_0$ can be substituted with the system equations of the leader and followers

$$\begin{aligned} \dot{S}(t) &= (L+B) \otimes I_n \left(U(t) - 1_N \otimes u_0(t) - U_2(t) \right) \\ &= (L+B) \otimes I_n \left(U_1(t) - 1_N \otimes u_0(t) \right) \end{aligned} \tag{10}$$

Let $U(t) = \left[u_1(t), u_2(t), \ldots, u_N(t) \right]^T$ and $U_1(t) = \left[u_{i1}(t), u_{i2}(t), \ldots, u_{N2}(t) \right]^T$. Substituting the equivalent control on the sliding surface U_1 input into equation , the equation of $\dot{S}(t)$ gives.

$$\dot{S}(t) = (L+B) \otimes I_n \left(\sigma sgn\left(S(t)\right) - 1_N \otimes \left(u_0(t) - u_{min}(t) \right) \right)$$

Select the Lyapunov functional as

$$V_s(t) = \frac{1}{2} S(t)^T \left(\left((L+B) \otimes I_n \right)^{-1} \right) S(t)$$

Along the sliding mode differential, $V_s(t)$ evolves as follows:

$$\begin{aligned} \dot{V}_s(t) &= S(t)^T \left((L+B) \otimes I_n \right)^{-1} \dot{S}(t) \\ &= S(t)^T \left((L+B) \otimes I_n \right)^{-1} (L+B) \otimes I_n \\ &\qquad \left(-\sigma sgn\left(S(t)\right) - 1_N \otimes \left(u_0(t) - u_{min}(t) \right) \right) \\ &= S(t)(-\sigma sgn\left(S(t)\right) - 1_N \otimes \left(u_0(t) - u_{min}(t) \right) \end{aligned}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202503_11(3).0002

Since $-\left(u_0(t)-u_{min}(t)\right)\le 0$, $\dot{S}(t)\le -\sigma sgn\left(S(t)\right)$ .Thus

$$\begin{aligned}\dot{V}_s(t) &= S(t)^T\left(\left(L+B\right)\otimes I_n\right)^{-1}\dot{S}(t)\\ &\le S(t)^T\left(-\sigma sgn\left(S(t)\right)\right)\\ &\le -\sigma \| S(t)\|\end{aligned}$$

For $t\in\left[t_{k_i}^i,t_{k_{i+1}}^i\right]$, we choose $\sigma\ge 0$. This means that when $S(t)=0$, $\dot{V}_s(t)<0$ .Therefore, we can confirm that the state of multiple drones will be forced to converge to the sliding mode surface constructed in equation (4) within a finite time. This constitutes the conclusion of the proof.

3.2 Consensus Analysis

This section analyzes the convergence issue of leader-follower consensus for multi-agent systems (1) and (2) under the effect of control protocol (6).When the state of the multi-UAV system enters the integral sliding manifold, we have $S=\dot{S}=0$. It means $\dot{V}(t)-1_N\otimes\dot{v}(t)=U_2(t)$ .Then, we can derive the sliding mode dynamics for $i=1,2,3,\ldots,n$ .Let $\dot{X}v_i(t)=v_i(t)-v_0(t)=u_{i2}(t)$ and $\dot{X}p_i(t)=p_i(t)-p_0(t)$.

The measurement error of UAV i and UAV j is given by:

$$\begin{aligned}e_{p_i}(t)&=p_i\left(t_k^i\right)-p_i(t)\\ e_{v_i}(t)&=v_i\left(t_k^i\right)-v_i(t)\\ e_{pij}(t)&=p_j\left(t_k^i\right)-p_j(t)\\ e_{vij}(t)&=v_j\left(t_k^i\right)-v_j(t)\end{aligned}\tag{11}$$

Respectively represent the position error and velocity error of UAV i and j within the interval , substituting into equation (8) yields.

$$\begin{aligned}u_{i2}(t) &= K_1\sum_{j\in\mathcal{N}_i}a_{ij}\left[p_j(t)-p_i(t)+v_j(t)-v_i(t)+e_{p_j}(t)-e_{p_i}(t)?+e_{v_j}(t)-e_{v_i}(t)\right]\\ &\quad -K_2a_{i0}\left[p_i(t)-p_0(t)+v_i(t)-v_0(t)+e_{p_i}(t)+e_{v_i}(t)\right]\end{aligned}\tag{12}$$

we have

$$\begin{aligned}\dot{X}_{p_i}(t) &= X_{v_i}(t)\\ \dot{X}_{v_i}(t) &= K_1\sum_{j\in\mathcal{N}_i}a_{ij}\left[p_j(t)-p_i(t)+v_j(t)-v_i(t)+e_{p_j}(t)-e_{p_i}(t)+e_{v_j}(t)-e_{v_i}(t)\right]\\ &\quad -K_2a_{i0}\left[p_i(t)-p_0(t)+v_i(t)-v_0(t)+e_{p_i}(t)+e_{v_i}(t)\right]\end{aligned}\tag{13}$$

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Let $\partial(t)=\left(X_p^T(t),X_v^T(t)\right)^T$, $X_p(t)=\left(X_{p_1}^T(t),X_{p_2}^T(t),\ldots,X_{p_n}^T(t)\right)^T$ $X_v(t)=\left(X_{v_1}^T(t),X_{v_2}^T(t),\ldots,X_{v_n}^T(t)\right)^T$ $X_v(t)=\left(X_{v_1}^T(t),X_{v_2}^T(t),\ldots,X_{v_n}^T(t)\right)^T$,and $e_p(t)=\left(e_{p_1}^T(t),e_{p_2}^T(t),\ldots,e_{p_n}^T(t)\right)^T$, $e_v(t)=\left(e_{v_1}^T(t),e_{v_2}^T(t),\ldots,e_{v_n}^T(t)\right)^T$, $e_p^{'}(t)=\left(e_{p_1}^T(t),e_{p_{12}}^T(t),\ldots,e_{p_{1n}}^T(t),\ldots e_{p_{n1}}^T(t),e_{p_{n2}}^T(t)\ldots,e_{pn}^T(t)\right)^T\in R^{n^2}$, $e_v^{'}(t)=\left(e_{v_1}^T(t),e_{v_{12}}^T(t),\ldots,e_{vn}^T(t),\ldots e_{v_{n1}}^T(t),e_{v_{n2}}^T(t)\ldots,e_{v_n}^T(t)\right)^T\in R^{n^2}$

We have

$$
\begin{aligned}
\dot{X}_p(t) &= X_v(t)\\
\dot{X}_v(t) &= -(K_1L+K_2B)\otimes I_m\left(X_p(t)+X_v(t)\right)\\
&\quad -K_1\tilde{L}\otimes I_m\left(e_p^{'}+e_v^{'}\right)-K_2B\otimes I_m\left(e_p+e_v\right)
\end{aligned}
$$

Where L is the picture G Laplace's matrix , $\tilde{L}=diag\left\{L_i\right\}(i=1,2,\ldots n)$, $\tilde{L}$ is the i row of the L matrix

Define the trigger function f_i as

$$
\begin{aligned}
f_i(e_{pi},e_{vi},\{e_{xij},e_{vij}\mid j\in\mathcal{N}_i\}) &= \sum_{j\in\mathcal{N}_i}(K_1\mid N_i\mid a_{ij}+K_2a_{i0})\left(\| e_{xi}\|^2+\| e_{vi}\|^2+\| e_{xij}\|^2+\| e_{vij}\|^2\right)\\
&\quad -\theta_i\alpha\mid N_i\mid\left[\lambda_{\min}(K_1\hat{L}+K_2B)-\alpha(2K_1\mid N_i\mid+a_{i0}k_2)-1\right]\\
&\quad \left(\| X_{pi}|^2+\| X_{vi}\|^2\right).
\end{aligned}
$$

Theorem 2 If the following conditions are true, the system can solve the leader following consistency problem,let $\lambda_{min}\left(K_1\hat{L}+K_2B\right)>1$, where $\hat{L}=\left(L+L^T\right)/2$.

Proof. First, Choose the d Lyapunov function

$$
V_c=\frac{1}{2}\partial^T(t)\left(P\otimes I_m\right)\partial(t) \tag{14}
$$

Derivation of 14 yields

$$
\begin{aligned}
\dot{V}_c &= X_p^T(K_1(L+L^T)+2K_2B)\otimes I_mX_v+X_v^TX_v+X_p^T\dot{X}_v+X_v^T\dot{X}_v\\
&=X_p^T(K_1(L+L^T)+2K_2B)\otimes I_mX_v++X_v^TX_v-X_p^T(K_1L+K_2B)\\
&\quad \otimes I_m\left(X_p+X_v\right)-X_v(K_1L+K_2B)\otimes I_m\left(X_p+X_v\right)-\left(X_p+X_v\right)\\
&\quad K_1\tilde{L}\otimes I_m\left(e_p^{'}+e_v^{'}\right)-\left(X_p+X_v\right)K_2B\otimes I_m\left(e_p+e_v\right)\\
&= -X_p^T\left(K_1\hat{L}+K_2B\right)\otimes I_mX_p+X_v^TX_v-X_v^T\left(K_1\hat{L}+K_2B\right)\otimes I_mX_v\\
&\quad -\left(X_p+X_v\right)K_1\hat{L}\otimes I_m\left(e_p^{'}+e_v^{'}\right)-\left(X_p+X_v\right)K_2B\otimes I_m\left(e_p+e_v\right)\\
&\le -\lambda_{min}\left(K_1\hat{L}+K_2B\right)\sum_j^n\left(\| X_p\|^2+\| X_v\|^2\right)+\sum_j^n\| X_v\|-\left(X_p+X_v\right)\\
&\quad K_1\tilde{L}\otimes I_m\left(e_p^{'}+e_v^{'}\right)-\left(X_p+X_v\right)K_2B\otimes I_m\left(e_p+e_v\right)
\end{aligned}
$$

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Then

$$
\begin{aligned}
&-\left(X_p+X_v\right)K_1\tilde{L}\otimes I_m\left(e_p^{'}+e_v^{'}\right)\\
&=-\sum_{i=1}^{n}\left(X_{p_i}^T(t)+X_{v_i}^T(t)\right)\left[K_1\sum_{j\in\mathcal{N}_{\spadesuit}}a_{ij}\left(e_{pi}+e_{vi}-e_{pij}-e_{vij}\right)\right]\\
&=-K_1\sum_{i=1}^{n}\left|N_i\right|\left(X_{p_i}^T(t)+X_{v_i}^T(t)\right)\left(e_{pi}+e_{vi}\right)\\
&\quad+\sum_{i=1}^{n}\left(\sum_{j\in\mathcal{N}_{\spadesuit}}\left(X_{p_i}^T(t)+X_{v_i}^T(t)\right)\left(e_{pij}+e_{vij}\right)\right)
\end{aligned}
$$

And

$$
\begin{aligned}
&-\left(X_p+X_v\right)K_2B\otimes I_m\left(e_p+e_v\right)\\
&=-\sum_{i=1}^{n}\left(X_{p_i}^T(t)+X_{v_i}^T(t)\right)K_2a_{i0}\left(e_{pi}+e_{vi}\right)
\end{aligned}
$$

Using Young's inequality $\left|xy\right|\le\frac{a}{2}x^2+\frac{1}{2a}y^2$ we have

$$
\begin{aligned}
&-K_1\sum_{i=1}^{n}\left(\sum_{j\in\mathcal{N}_{\spadesuit}}a_{ij}\left(X_{p_i}^T(t)+X_{v_i}^T(t)\right)\right)\left(e_{pi}+e_{vi}-e_{pij}-e_{vij}\right)\\
&\le\alpha\sum_{i=1}^{n}K_2a_{i\ 0}\left(\|X_{p_i}\|^2+\|X_{v_i}\|^2\right)+\frac{1}{\alpha}k_2a_{i0}\left(\|e_{p_i}\|^2+\|e_{v_i}\|^2\right)
\end{aligned}
$$

we can obtain

$$
\begin{aligned}
\dot{V}_c\ \le\ \sum_{i=1}^{n}\ &\left\{\left[\lambda_{min}\left(K_1\hat{L}+K_2B\right)+\alpha\left(2K_1\left|N_1\right|+K_2a_{i0}+1\right)\right]\right.\\
&\left(\|X_{p_i}\|^2+\|X_{v_i}\|^2\right)+\frac{K_1}{\alpha}\sum_{j\in\mathcal{N}_{\spadesuit}}\left(K_1a_{ij}+\frac{K_2a_{i0}}{\left|N_i\right|}\right)\\
&\left.\times\left(\|e_{pi}\|^2+\|e_{vi}\|^2+\|e_{pij}\|^2+\|e_{ij}\|^2\right)\right\}
\end{aligned}
$$

Where $0\le\alpha\le\frac{\lambda_{min}\left(K_1\hat{L}+K_2B\right)-1}{2K_1\left|N_i\right|K_2a_{i0}}$ By f_i, we have

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$$\dot{V}_c \leq -\sum_{i=1}^{n}(1-\theta_i)\Big[\lambda_{min}\left(K_1\hat{L}+K_2B\right)-\alpha\left(2K_1\left|N_i\right|+a_{i0}K_2\right)-1\Big]\left(\| X_{p_i}\|^2+\| X_{v_i}\|^2\right)<0$$

where $0<\theta_i<1$.

According to Lyapunov stability theory, for any initial state, the solution of the equation of state satisfies the consistency definition, so the system solves the leader-following consistency problem.

In order to show that Zeno phenomenon does not occur during the whole event triggering control process

Theorem 3 Considering multiple unmanned aerial systems (1) and (2) under the action of consistency protocol (6), there is at least one unmanned aerial vehicle p whose interval $t_{k+1}^{q}-t_{k}^{q}$ between any two consecutive event triggering moments is not less than

$$\tau_p=\frac{1}{\sqrt{2m}\,\| K_1L+K_2B\|+\sqrt{mn}}\frac{M_q}{\sqrt{2n}\,\|\Lambda\|+M_q}$$

Where $\Lambda=diag\left\{\Lambda_i\right\}\left(i=1,2,\ldots n\right)$, Λ_i is row i of the matrix $D+A$,and.

$$M_q=\left[\theta_q\alpha\left|N_q\right|\frac{\lambda_{min}\left(K_1\hat{L}+K_2B\right)-\alpha\left(2K_1\left|N_p\right|+K_2a_{q0}\right)-1}{K_1\left|N_q\right|+K_2a_{q0}}\right]^{\frac{1}{2}}$$

Proof. First, by Theorem 3 ,we have

$$\sum_{i=1}^{n}\left(\sum_{j\in\mathcal{N}_{\blacklozenge}}\left(e_{pi}^2+e_{vi}^2+e_{pij}^2+e_{vij}^2\right)\right)\leq\left\|\begin{bmatrix}\Lambda\\ \Lambda\end{bmatrix}\right\|^2\| e'\|^2$$

If $q=arg\ max\left(\| X_{p_i}\|^2+\| X_{v_i}\|^2\right)$, then there is

$$\frac{\sum_{j\in\mathcal{N}_q}\left(\| e_{pq}\|^2+\| e_{vq}\|^2+\| e_{pqj}\|^2+\| e_{vqj}+\|^2\right)}{n\left(\| X_p\|^2+\| X_v\|^2\right)}\leq\frac{2\|\Lambda\|^2\| e'\|^2}{\|\partial\|^2}$$

Then

$$\sqrt{2n}\,\|\Lambda\|\frac{\left(\sqrt{2m}\,\| K_1L+K_2B\|+\sqrt{mn}\right)\tau_q}{1-\left(\left(\sqrt{2m}\,\| K_1L+K_2B\|+\sqrt{mn}\right)\tau_q\right)}$$

$$=\Big[\theta_q\alpha\left|N_q\right|\times\frac{\lambda_{min}\left(K_1\hat{L}+K_2B\right)-\alpha\left(2K_1\left|N_q\right|+K_2a_{q0}\right)-1}{K_1\left|N_q\right|+K_2a_{q0}}\Big]^2$$

we can obtain

$$\tau_p = \frac{1}{\sqrt{2m}\,\|K_1L + K_2B\| + \sqrt{mn}}\frac{M_q}{\sqrt{2n}\,\|\Lambda\| + M_q}$$

4. Simulation

This section demonstrates a simulation example of distributed multi-UAV cooperation to verify the practicality and effectiveness of the proposed distributed event-triggered sliding mode control scheme. In this scenario, one leader and five followers are considered. The communication topology is shown in Fig. 1. For ease of operation, considering only the case of $m=1$, the easy Laplacian matrix to G and the leading pair adjacency matrix are respectively $B = diag(1,0,1,0,0)$ and

$$L = \begin{pmatrix} 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 1 \end{pmatrix}.$$

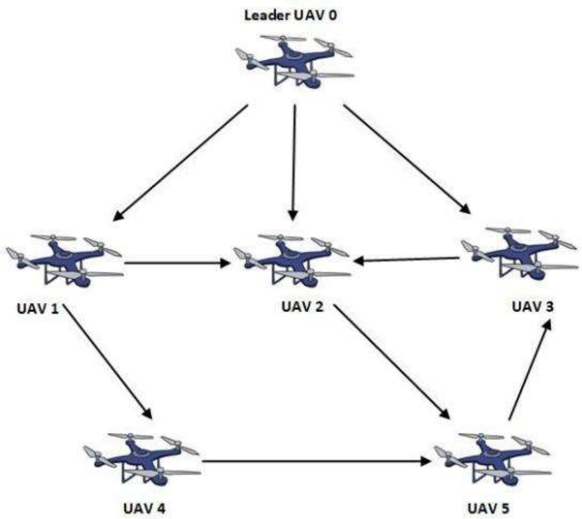

Fig. 1 Communication topology

ByTheorem2, we can obtain $\sigma = 0.1, \theta = 0.99$ We select $K_1 = 1.5, K_2 = 2$, $\alpha = 0.05$, The leader’s control can choose that $u_{max} = 1$ and $u_{min} = 1$ .Fig. 2 and Fig. 3 respectively show the lead following position trajectory and speed trajectory of the multi-agent system, and Fig. 4 and Fig. 5 respectively show the change trend of the lead following position and speed difference of the multi-agent system. Fig.5 shows the event trigger controllers for six UAVs.Therefore, we can clearly see from the figure above that the designed distributed event sliding mode controller can ensure the UAV tracking control and effectively reduce the waste of communication resources.

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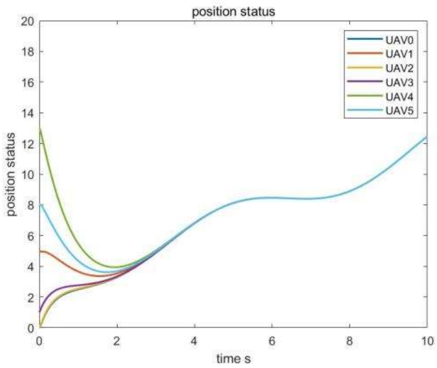

Fig. 2 Positional states

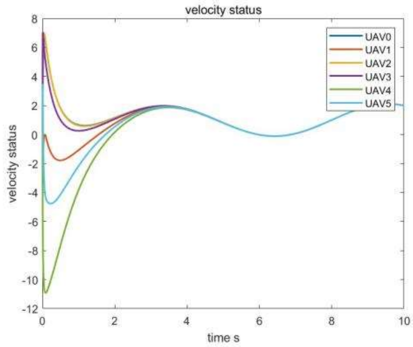

Fig. 3 Velocity states

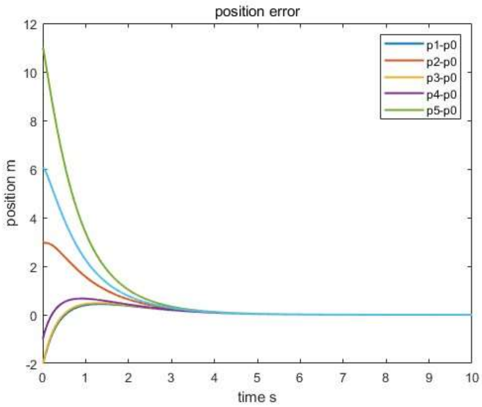

Fig. 4 Position error

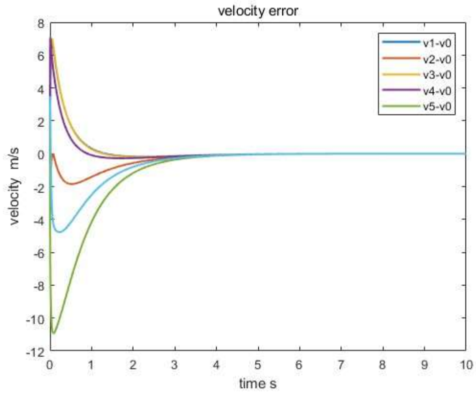

Fig. 5 Velocity error

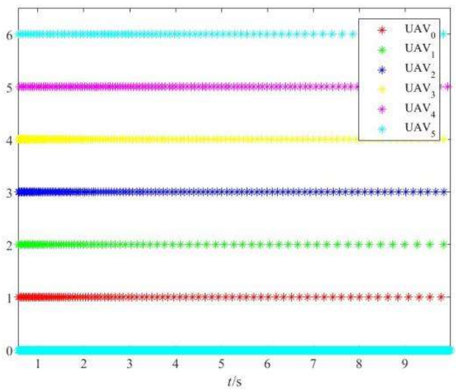

Fig. 6 Event-triggering interva

5. Conclusion

The master-slave consistency problem of second-order multi-UAVs is studied. The distributed sliding mode control protocol proposed in this paper effectively solves the problem of the leader receiving unknown dynamic control inputs. At the same time, considering that there are some unavoidable situations in the communication transmission between various UAVs in the actual control system, it is suggested to choose the event triggering control protocol, which can effectively avoid Zeno behavior by designing appropriate event triggering function. Finally, the effectiveness of the proposed control protocol is verified by simulation. The proposed UAV formation has been verified by virtual simulation, but the real conditions are dynamic and changeable. The following work will further investigate the consistency of multi-agent systems in switching topology and with external interference

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