Academic Journal of Law and Society
ISSN: 3080-9908 (Print), ISSN: 3080-9916 (Online) | Vol. 1, No. 1, 2026

Research on Selection of Engineering Coordinate System and Coordinate Transformation Method for High-Speed Railway

Wenlong Zhou

School of Surveying and Land Information Engineering., Henan Polytechnic University, Jiaozuo 454000, Henan, China

Abstract: In view of the engineering characteristics of high-speed railway long and narrow linear projects, such as large span, significant topographic relief, and strict control of projection deformation, the traditional national unified coordinate system can hardly meet the accuracy requirement that the projection length deformation is ≤ 2.5 cm/km. Taking high-speed railway engineering survey as the research background, this paper sorts out the core theories and selection principles of engineering coordinate systems, constructs mathematical models including ellipsoid transformation and Bursa seven-parameter transformation, and carries out projection deformation calculation and coordinate system optimization design with examples of Taibai-Fengxian Expressway and typical mountainous railway sections. The results show that by customizing the compensation elevation plane and adjusting the position of the central meridian, the elevation deformation and Gauss projection deformation can be effectively compensated, and the global deformation value can be controlled within 1/43000, which meets the accuracy specifications for high-speed railway engineering survey. The coordinate transformation method based on the seven-parameter model enables the accurate connection between the national coordinate system and the independent engineering coordinate system. The coordinate system selection strategy and transformation method proposed in this paper can provide practical references for the coordinate system design of similar linear projects.

Keywords: High-speed railway; Engineering coordinate system; Coordinate transformation; Projection deformation; Bursa model.

1. Introduction

In recent years, China's high-speed railway construction has developed rapidly. By the end of 2025, the national high-speed railway operating mileage exceeded48,000 kilometers, ranking first in the world and becoming the core support for building a country with strong transportation network. As a typical long and narrow linear project, a single high-speed railway line often spans dozens to thousands of kilometers, and mostly passes through complex landforms such as mountains, hills and plateaus, with topographic relief of up to several hundred meters. This imposes far stricter requirements on the accuracy, stability and consistency of engineering surveying than ordinary projects. According to the Code for Railway Engineering Survey (TB 10101-2018) and Specifications for Highway Reconnaissance (JTG C10-2007), the projection length deformation in horizontal control survey of high-speed railways shall not exceed 2.5 cm/km, and for key large-scale structures such as bridges and tunnels, it shall be further strictly controlled within 1 cm/km to ensure the accuracy of line setting-out and track laying[1].

However, the traditional national unified coordinate systems adopted in surveying and mapping, such as the Beijing 1954 Coordinate System and the Xi'an 1980 Coordinate System, are designed with fixed Gauss zones (3° zones or 6° zones) and a single reference ellipsoid, without considering elevation fluctuations and geographical differences in local areas[2]. In high-speed railway sections with high altitude, cross-zone distribution and large topographic relief, the elevation deformation caused by reducing ground distances to the reference ellipsoid and the Gauss projection deformation from projecting the ellipsoid onto the plane will be superimposed and amplified. In some areas, the total deformation can reach up to one part in several thousand, far exceeding the specification limits. This directly leads to distortion of survey results and deviations in line setting-out, seriously affecting the construction quality and operational safety of high-speed railway projects[3]. Therefore, establishing a customized engineering coordinate system suitable for the characteristics of high-speed railway projects, accurately controlling projection deformation, and realizing seamless conversion among multiple coordinate systems has become a key technical problem urgently to be solved in high-speed railway engineering survey. As a customized surveying and mapping datum system adapted to the geographical location, elevation characteristics and project scope, the engineering coordinate system realizes accurate compensation of projection deformation by adjusting reference ellipsoid parameters, optimizing the position of the central meridian, setting a compensation projection plane and other methods, which is the core measure to ensure the survey accuracy of high-speed railways[4]. At present, scholars at home and abroad have carried out extensive research on coordinate system conversion and projection deformation control. Foreign research started relatively early, focusing on the optimization of high-precision conversion algorithms for geocentric coordinate systems. Classical three-dimensional conversion models such as the Bursa model, the Molodenskii model and the Vanicek model have been successively proposed, emphasizing the unified adaptation of global coordinate systems. Domestic research is more in line with local engineering demands, focusing on the establishment of local independent coordinate systems, the improvement of ellipsoid transformation methods and deformation control strategies for linear projects. Nevertheless, the deformation complementary mechanism, standardized coordinate system selection process and multi-segment coordinate system connection methods for extra-long linear, cross-zone and high-drop projects such as high-speed railways still need to be improved. Most existing studies focus on the application of a single model and lack a systematic whole-process scheme covering selection, modeling, verification and application[5-6].

Based on the above analysis, this paper takes high-speed railway engineering survey as the core research object. Combined with the characteristics of high-speed railway linear projects, it systematically sorts out the basic theories and selection specifications of engineering coordinate systems, and constructs mathematical models including ellipsoid translation, ellipsoid expansion, and Bursa seven-parameter transformation. Relying on program calculation, two typical cases—a mountainous high-speed railway section and the Taibai–Fengxian Expressway—are selected for empirical research, to quantitatively analyze the projection deformation control effect after coordinate system optimization, and summarize high-speed railway coordinate system selection strategies suitable for different working conditions. This study aims to improve the theoretical system of coordinate system design for high-speed railway engineering, provide replicable practical schemes for coordinate system selection and coordinate transformation of similar ultra-long linear projects, and further enhance the accuracy, practicality and compatibility of high-speed railway engineering survey results.

2. Study Area Overview and Data Sources

2.1. Study Area Overview

This paper selects two typical linear projects as the study areas, namely a mountainous high-speed railway section and the Taibai–Fengxian Expressway. The study areas cover complex working conditions such as mountainous terrain and cross-zone regions, which correspond to the core difficulties in high-speed railway engineering survey.

The typical mountainous high-speed railway section is located around 29°56′N latitude and 106°10′E–106°50′E longitude, generally extending from northwest to southeast. The main line is 70 km long, with an east-west span of 30 km and a north-south width of 8 km in the survey area, forming a typical long and narrow strip. This section is situated in the mountainous area on the southern foot of the Qinling Mountains, with rugged surface and crisscross gullies. The overall elevation is approximately 300 m. The designed track elevation has a minimum of 220 m in the east and a maximum of 260 m in the west, with a vertical relief of 40 m. Moreover, the section lies exactly across the boundary of the 35th and 36th zones in the 3° Gauss-division scheme. Obvious zone-joint deviations and projection distortions exist in the national unified coordinate system in this area, with the measured deformation reaching up to 1/20000 (5 cm/km), far exceeding the requirements of high-speed railway survey specifications. Therefore, it is necessary to establish a customized engineering coordinate system to meet the accuracy requirements.

Taibai–Fengxian Expressway: As a regional arterial road connecting Shaanxi and Gansu provinces, the expressway has a total length of 54 km, starting from Tianma Intersection in Fengxian County in the west and ending at the Shaanxi-Gansu border in the east, with an overall east-west alignment. The survey area is located between 33°50′N–33°58′N latitude and 106°30′E–106°55′E longitude. This section lies in the transition zone between the Loess Plateau and the Qinling Mountains, with a terrain that is low in the west and high in the east and significant topographic relief. The elevation of the starting point GN28 is 966.728 m, and that of the ending point GP16 is 1272.510 m. The average elevation along the whole line is about 1100 m, with a maximum elevation difference exceeding 300 m. In addition, it is situated at the edge of the 3° zone of the Xi'an 1980 Coordinate System. The projection deformation of the traditional coordinate system shows a distribution characteristic of small in the west and large in the east. The deformation in some western areas exceeds the limit of 2.5 cm/km, making it unsuitable for direct engineering surveying, mapping and construction setting-out.

2.2. Data Sources

The data sources used in this study include three categories: basic surveying and mapping data, coordinate system parameters, and engineering measured results. All data are from authoritative sources, and their accuracy meets the research requirements, which are detailed as follows:

(1) Coordinate system parameters: Beijing 1954 Coordinate System (Krasovsky ellipsoid, semi-major axis 6378245 m, flattening 1/298.3); Xi'an 1980 Coordinate System (International 1975 ellipsoid, semi-major axis 6378140 m, flattening 1/298.257); and WGS84 geocentric coordinate system parameters.

(2) Measured coordinate data: 5 sets of common geocentric coordinate data of WGS84 and Beijing 1954 Coordinate System, including spatial rectangular coordinates X, Y, and Z values; geodetic coordinates, Gauss plane coordinates, and elevation data of the start and end points of the Taibai–Fengxian Expressway; Digital Elevation Model (DEM) data of the study area and height anomaly data of the gravity field model.

(3) Specification data: Mandatory provisions on projection deformation control, coordinate system selection, and coordinate transformation accuracy evaluation in the Code for Railway Engineering Survey and Specifications for Highway Reconnaissance; core mathematical formulas and derivation processes such as Gauss projection forward and inverse calculation, ellipsoid transformation, least squares adjustment, and Bursa model solution; and universal standards for coordinate transformation accuracy evaluation and error analysis methods in the surveying and mapping industry.

3. Research Methods

This paper carries out research around the technical route of "projection deformation mechanism analysis, coordinate system selection, mathematical model construction, coordinate transformation, and case verification". The core methods include four modules: basic theory of coordinate system, projection deformation control, ellipsoid transformation, and coordinate transformation model, which are detailed as follows.

3.1. Basic Theory of Engineering Coordinate System

3.1.1. Types of Commonly Used Coordinate Systems

Commonly used coordinate systems in engineering surveying are divided into four categories: national unified coordinate system, geocentric coordinate system, Gauss plane rectangular coordinate system, and local independent coordinate system. Each type of coordinate system has significant differences in definition, parameter characteristics, applicable scenarios and limitations, which are the basic basis for the selection of engineering coordinate systems: ① WGS84 geocentric coordinate system: an internationally used GPS positioning coordinate system with the origin at the Earths center of mass, adopting WGS84 ellipsoid parameters (semi-major axis 6378137 m, flattening 1/298.257223563). It is suitable for global satellite positioning and 3D coordinate acquisition, and serves as the basic datum for modern engineering surveying; ② Beijing 1954 Coordinate System: an early geocentric coordinate system in China, adopting the Krasovsky ellipsoid, which is suitable for national basic surveying and mapping. However, the ellipsoid parameters have poor fit with Chinas terrain, and deformation is obvious in high-altitude areas; ③ Xi'an 1980 Coordinate System: Chinas second -generation geocentric coordinate system, adopting the International 1975 ellipsoid with the geodetic origin in Jingyang, Shaanxi Province. It is suitable for national surveying and mapping operations and is the mainstream coordinate system for traditional engineering surveying; ④ Gauss plane rectangular coordinate system: taking the central meridian as the vertical axis X and the equator as the horizontal axis Y, it converts ellipsoidal coordinates into plane coordinates through Gauss projection. It is convenient for calculation and suitable for engineering setting-out, and is the core projection coordinate system for engineering surveying; ⑤ Local independent coordinate system: optimized and improved based on the national coordinate system, it minimizes projection deformation in local areas by adjusting ellipsoid parameters, projection plane and central meridian. Customized for specific projects, it has extremely strong accuracy adaptability[7].

3.1.2. Principles for Selecting Coordinate Systems

High-speed railways are typical linear projects. Especially in mountainous areas or when the line crosses multiple national unified 3° zones, it is difficult for the side length projection deformation to meet the requirement of 2.5 cm/km. To achieve good compensation between the two types of deformation, it is necessary to move the central meridian and reselect a new elevation plane. Let the geodetic height of the reselected elevation reference plane be H, the elevation of the distance-measuring side relative to the new elevation reference plane be ΔH, and the average value of the abscissas of the two endpoints of the distance-measuring side relative to the reselected central meridian be y. Then the conditional formula that meets the requirements of high-speed railways for projection deformation can be approximately expressed as:

$$\left|\frac{y^2}{2R_m^2}-\frac{\Delta H}{R_A}\right| \le \frac{1}{40000} \qquad (1)$$

Expanding the above formula, we can get:

$$\left(+\frac{y^2}{2R_m}-\frac{R}{40000}\right) \le \Delta H \le \left(+\frac{y^2}{2R_m}+\frac{R_A}{40000}\right) \qquad (2)$$

In the formula, R_A is the curvature radius of the normal section on the reference ellipsoid for the reduced line direction; Rm is the average curvature radius of the midpoint of the ranging line. Based on the constraint conditions of Equation (2), the design of the arbitrary zone coordinate system for the compensated elevation plane can be carried out. According to the content of this chapter, there are two methods to control the length deformation:(1) Select an appropriate elevation projection plane, namely, adopt the compensated elevation plane;(2) Try our best to make the central meridian of the zone division located at the center of the survey area.

3.2. Ellipsoid Transformation Method

By adjusting the parameters of the national reference ellipsoid to construct a compensated coordinate system, two core methods are adopted: the ellipsoid translation method and the ellipsoid expansion method.

3.2.1. Ellipsoid Translation Method

The principle of the ellipsoid translation method is as follows:Initially, unconstrained and constrained GPS network adjustment is performed to obtain the coordinates of the national geodetic survey dataset. Subsequently, ellipsoid transformation is utilized to convert and project these coordinates onto the plane of a reference ellipsoid defined at a given elevation plane.Finally, projection transformation is carried out via the central meridian transformation to obtain the coordinates in the independent coordinate system.Figure 1 illustrates the workflow of this ellipsoid translation method[8].

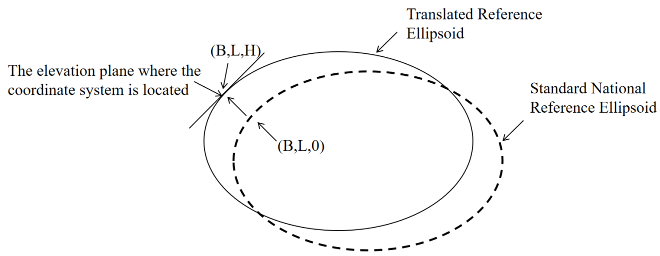

Fig 1 Ellipsoid Translation Method

Step 1: Determine the latitude B and longitude L within the project area.

Step 2: Obtain the height anomaly value & of the project center area from the Earths gravity field model.

Step 3: Calculate the geodetic height H of the projected elevation (H=h+height anomaly value &), that is, add the determined normal height h of the projected elevation plane to the height anomaly calculated above.

Step 4: Convert the geodetic coordinates (B, L, H) of the project center area into spatial rectangular coordinates (X, Y, Z).

Step 5: Calculate the spatial rectangular coordinates (X, Y, Z) of the center area, and translate the ellipsoid along the normal direction of the center area so that the surface of the ellipsoid is tangent to the required elevation plane.

Step 6: Calculate the translation value of the ellipsoid through the original spatial rectangular coordinates (X, Y, Z) and the translated spatial rectangular coordinates (X, Y, Z).

Step 7: Use the sum of the calculated displacement amounts to convert the 3D coordinates of the original coordinate system into the translated coordinate system, and then project from the new coordinate system to obtain the coordinates of the new coordinate system.

Note: When using the ellipsoid offset method (i.e., ellipsoid translation method), there will be a significant numerical difference between the plane coordinates of a specific topographic surface and the plane coordinates of the national reference ellipsoid surface, even though the geometric relationship between each point remains unchanged.

3.2.2. Ellipsoid Expansion Method

Principle of the ellipsoid expansion method: First, draw an unconstrained and unrestricted GPS network, obtain coordinates from the national geodetic survey database, then project the coordinates onto a reference ellipsoid at a certain height through ellipsoid transformation, and finally convert the central meridian for projection transformation to obtain the coordinates of the independent coordinate system[9].

The ellipsoid is located above the average elevation plane, with its center, axis and flattening consistent with the national reference ellipsoid, but its semi-major axis has changed. The ellipsoid expansion method is shown in Figure 2 below.

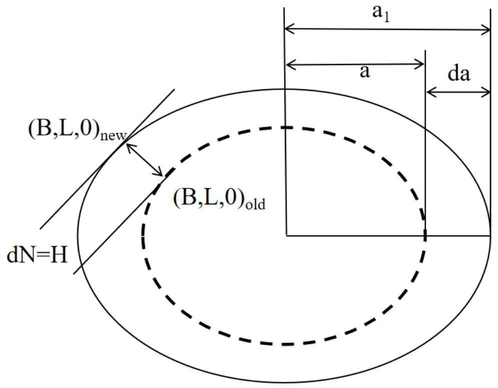

Fig 2 Ellipsoid Expansion Method

(1) First, determine the latitude *B* and longitude *L* in the area of the projection center.

(2) Obtain the height anomaly value *&* of the projection center area, which is derived from the Earths gravity field model.

(3) Add the specified height *h* of the projected elevation plane to the height anomaly obtained above to get the geodetic height *H* of the projected elevation plane.

(4) Calculate the variation of the semi-major axis of the new ellipsoid.

(5) Calculate the semi-major axis of the new ellipsoid: $a_1=a+\Delta a$, where a is the maximum radius of the national reference ellipsoid, and Δa is the variation of the semi-major axis.

(6) Convert all coordinates within the original datum plane into three-dimensional rectangular coordinates, and project the three-dimensional rectangular coordinates onto the new ellipsoid used as the datum plane to obtain the plane coordinates within the height defined by the new ellipsoid coordinate system.

Note: If the ellipsoid expansion method is adopted, the plane coordinates obtained after projection are numerically consistent with the plane coordinates within the national reference ellipsoid plane, but only scaled appropriately at a suitable scale.

3.3. Coordinate Transformation Model

3.3.1. Bursa Seven-Parameter Model

Principle: First, perform unconstrained adjustment of the GPS network to obtain the WGS84 coordinates under the geocentric static system. Then, obtain the coordinates of the common points in the specified and projected independent elevation coordinate system. Next, use the WGS84 coordinates and the coordinates of the independent common coordinate system of these common points to solve the seven parameters of the Bursa model. Finally, convert the WGS84 coordinates of all points into the coordinates of the independent coordinate system of the model. This model converts the WGS84 coordinates of all points into an independent coordinate system.

The Bursa model is also known as the seven-parameter transformation model. Its mathematical model is as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + uR(\varepsilon X)R(\varepsilon Y)R(\varepsilon Z)\begin{bmatrix} x \\ y \\ z \end{bmatrix} \tag{3}$$

In Equation (3), $[X \quad Y \quad Z]^T$ is the transformed coordinate, $[x \quad y \quad z]^T$ is the coordinate to be transformed, $[\Delta X \quad \Delta Y \quad \Delta Z]^T$ is the translation amount, and u is the scale parameter.

In general, εX、εY、εZ is a small angle, so the first-order term approximation of Euler angles can be adopted, $\cos\varepsilon$ is approximately equal to 1, and $\sin\varepsilon = \varepsilon$. Then we have:

$$R(\varepsilon X)R(\varepsilon Y)R(\varepsilon Z) = \begin{bmatrix} 1 & \varepsilon Z & -\varepsilon Y \\ -\varepsilon Z & 1 & \varepsilon X \\ \varepsilon Y & -\varepsilon X & 1 \end{bmatrix} \tag{4}$$

By deformation and arrangement, we can get:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & -z & y & x \\ 0 & 1 & 0 & z & 0 & -x & y \\ 0 & 0 & 1 & -y & x & 0 & z \end{bmatrix}\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \varepsilon X \\ \varepsilon Y \\ \varepsilon Z \\ u \end{bmatrix} \tag{5}$$

For the above transformation, at least 7 equations are required to solve the 7 parameters in the model. Therefore, at least 3 control points must be jointly measured in the network, and generally, more than 3 control points are used to perform least squares solution to list the error equations.

$$V = AX - L \tag{6}$$

Solve using the least squares method:

$$X = (A^TPA)^{-1}(A^TPL) \tag{7}$$

Starting from the WGS84 geodetic coordinates and the corresponding coordinates of the independent coordinate system, solve the transformation parameters according to the above formula, and then convert the WGS84 3D coordinates of all other control points into the coordinates of the Bursa model independent coordinate system based on the solved parameters.

3.3.2. Plane Four-Parameter Transformation

For the coordinate transformation between the national coordinate system and the engineering plane coordinate system, a four-parameter model is adopted. This model includes two translation parameters, one scale parameter, and one rotation parameter. It can be solved based on two common points and is suitable for rapid on-site engineering transformation.

4. Results and Analysis

4.1. Coordinate System Optimization Results of a Typical Railway Section

Aiming at the problem of excessive deformation in cross-zone mountainous high-speed railway sections, this study adopts the optimization scheme of the compensated arbitrary zone Gauss plane coordinate system, and carries out refined design combined with the elevation and linear characteristics of the survey area: the geodetic height of the projection plane is determined to be 240m (consistent with the average elevation of the rail top) to offset the shortening deformation caused by elevation reduction; the central meridian is selected as 106°20′ (west side of the survey area) to reduce the abscissa deviation distance and the elongation deformation of Gauss projection. Through program calculation, the entire east-west width of the survey area under this coordinate system is 35km, which is completely within the deformation controllable range. The comprehensive projection deformation values of each control point are evenly distributed, the maximum comprehensive projection deformation value is 1/43000, which is far lower than the limit of 1/40000 (2.5cm/km) specified in the high-speed rail standard, and the minimum deformation value is only 1/80000. The deformation control effect is excellent, which fully meets the accuracy requirements of high-speed rail plane control survey and track setting-out.

4.2. Accuracy Results of Seven-Parameter Coordinate Transformation

To verify the transformation accuracy of the Bursa seven-parameter model, this study selected 5 groups of uniformly distributed and accurately coordinated common points between the WGS84 coordinate system and the Beijing 1954 coordinate system in the survey area. Among them, points 1-3 were used as known common points to solve the seven parameters, and points 4-5 were used as check points to verify the transformation accuracy.

Commonly used coordinate systems in engineering surveying include the Beijing 1954 coordinate system, the Xian 1980 coordinate system, and the independent coordinate system, etc. In the selection of engineering coordinate systems, it is necessary to consider the projection deformation between coordinate systems and the coordinate transformation between different systems. A summary of the corresponding coordinate transformations is provided below. The specific design is as follows:

Given the coordinates of 5 points in both the WGS84 coordinate system and the Beijing 1954 coordinate system, points 1-3 were selected to calculate the seven parameters of the 3D coordinate transformation. In addition, the WGS84 coordinates of points 4 and 5 were converted into the Beijing 1954 coordinate system.

Table 1 below shows the coordinates of the 5 points:

Table 1. Coordinate Data

Point Number	X84	Y84	Z84	X54	Y54	Z54
1	-2066241.5001	5360801.8835	2761896.3022	-2066134.4896	5360847.0595	2761895.5970
2	-1983936.0407	5430615.7282	2685375.7214	-1983828.7084	5430658.9827	2685374.6681
3	-1887112.7302	5468749.1944	2677688.9806	-1887005.1714	5468790.6487	2677687.2680
4	-1808505.4212	5512502.2716	2642356.5720	-1808397.7260	5512542.0921	2642354.4550
5	-1847017.0670	5573542.7934	2483802.9904	-1846909.0036	5573582.6511	2483801.6147

Note: All the above are geocentric coordinates.

① Calculate the Seven-Parameters for 3D Coordinate Transformation

To solve the seven parameters, we can obtain the coefficient matrix *B* according to the formula.

$$B=\begin{bmatrix}1 & 0 & 0 & 0 & -z & y & x\\ 0 & 1 & 0 & z & 0 & -x & y\\ 0 & 0 & 1 & -y & x & 0 & z\end{bmatrix} \quad (8)$$

Where the *x*, *y*, and *z* coordinates are the source coordinates (WGS84), next calculate the constant matrix *L*:

$$L=\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}-\begin{bmatrix}x\\ y\\ z\end{bmatrix} \quad (9)$$

Table 2. Transformed Results and Coordinates

Unit Weight Mean Error /m	Beijing 1954 Coordinates of Point 4	Beijing 1954 Coordinates of Point 5	Estimated Values of Seven-Parameter	Residual
0.028331	1.8084e+06	-1.8469e+06	-8.9459	-0.011015
	5.5125e+06	5.5736e+06	25.758	-0.0074235
	2.6424e+06	2.4838e+06	13.634	0.0066485
			2.7752e-06	0.028971
			-6.0234e-06	0.012864
			1.6864e-05	-0.0050207
			-4.3091e-06	-0.017956
				-0.0054405
				-0.0016277

In *XYZ*, which are Beijing 54 coordinates, and *xyz* are WGS84 coordinates, to find the corresponding constant matrix, next calculate the estimated value of the seven-parameter transformation matrix $\hat{X}=(B^TB)^{-1}(B^TL)$, and the obtained seven-parameter estimated value is a 7*1 order matrix. And the corresponding seven parameters (3 translation parameters, 3 rotation parameters (Euler angles), 1 scale parameter). Next, perform accuracy evaluation, find the residual *V*. According to the problem, the necessary number of observations t=7, n=9, r=n-t=2. Select the seven transformation parameters as estimated parameters, then calculate the unit weight mean error:

$$\hat{\sigma}_0=\sqrt{V^TV/n-t}=\sqrt{V^TV/9-7}=0.028m=28mm \quad (10)$$

② Convert the WGS84 coordinates of Points 4 and 5 into Beijing 1954 coordinates.

Calculate the transformed coordinates according to Formula 5, and then the Beijing 1954 coordinates of Points 4 and 5 can be obtained, as shown in Table 2.

4.3. Application Results of Taibai to Fengxian Expressway Project

The Taibai to Fengxian Expressway runs from Taibai County to Fengxian County. It intersects with the Shanan Expressway at Tianma Road Intersection in Fengxian County, and continues westward to connect with the Liangdu Expressway at the Shaanxi-Gansu border in Gansu Province. This project is divided into two design sections, responsible for the design and survey work of the 54-kilometer section from K33 to K87, which starts from Tianma Road Intersection in the west and extends to the Shaanxi-Gansu border via Fengxian County, as well as the east-west section of the new line, as shown in Table 3.

Table 3. Route Start/End Point Coordinates and Elevation Table

Name	Point Name	X(m)	Y(m)	B	L	H(m)
Start Point	GN28	3755248.244	354496.924	33°54′51″	106°25′36″	966.728
End Point	GP16	3761551.130	404516.948	33°58′36″	106°58′00″	1272.510

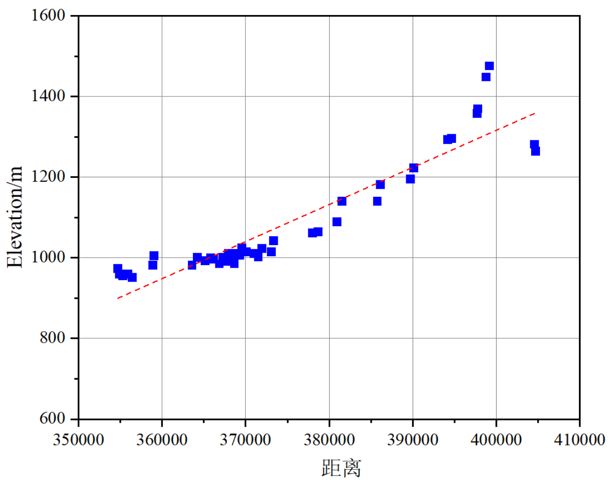

Fig 3 Elevation Change Trend Chart of the Route

The topographic elevation of the route from west to east is shown in Figure 3. From the perspective of the elevation trend, the elevation generally increases from west to east. The topographic distribution in this area extends about 15 kilometers eastward and westward from the starting point, with the slope gradually increasing. Then there are two hills to the east, reaching the mountain top about 5 kilometers away from the end of the area, and then descending more steeply. According to the above elevation changes on the map, the deformation value caused by the elevation change is calculated using the following formula.

$$\delta_d = \frac{-H_m}{R_m} D \tag{11}$$

In the above formula: δ_d is the length deformation value (caused by elevation); D is the length of the projected edge; Hm is the average elevation of the projected edge above the reference ellipsoid; Rm is the average curvature radius of the reference ellipsoid. The trend chart of the length deformation value caused by elevation change is shown in Figure 4.

It can be seen from Figure 4 that the length deformation caused by elevation change mainly increases from west to east (i.e., the deformation value increases). The minimum elevation at the western end of the survey section is 966.728 meters, with a length deformation value of 15.17 centimeters; the elevation at the easternmost end is 1272.510 meters, with a length deformation value of 19.97 centimeters. The calculated average elevation is approximately 1100 meters. The average elevation of 1100 meters in the survey area is used as the compensated elevation projection plane. The western side is about 150 meters below the plane, and the eastern side is 150 meters above the plane. The length deformation value calculated by Formula 11 is 2.4 centimeters per kilometer, which is less than the specification requirement that the projection length deformation value shall not exceed 2.5 centimeters per kilometer. If the central meridian is selected as the center of the survey area with a length of 50 kilometers, the calculation shall be carried out according to Formula 12:

$$\delta_d = \frac{Y_m^2}{2R_M^2} * D \tag{12}$$

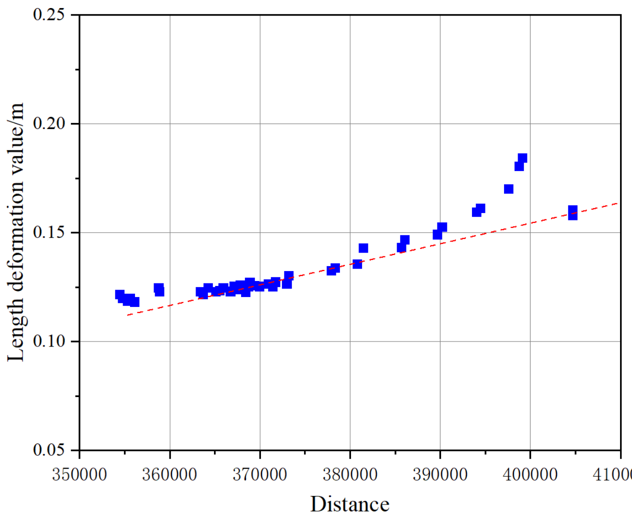

Fig 4 Trend Chart of Length Deformation Value Change Caused by Elevation Change

When calculating from the farthest points from the central meridian (the start and end points), the obtained length deformation value is 0.8 cm/km. Since the deformation value caused by elevation change is 2.4 cm/km, the difference between these two deformations is only 1.6 cm/km, which seems to comply with the specifications. However, it should be noted that the projected area in the west is smaller than that of the elevation projection. The deformation caused by high-elevation projection cannot be corrected by the error between the projection away from the central meridian and that at the central meridian. However, their combined effect makes the deformation in the western part of the survey area, especially at the western end, exceed the limit. Therefore, in the conventional method, the projection area of the survey area is assumed to be the average sea level of 1100 meters, and the central meridian of the survey area is taken as the central meridian.

In this project, we have solved this problem in detail as follows. From the trend of length deformation caused by elevation change, the deformation value caused by east-west elevation change in this area gradually increases. This can be compensated by the rule that "the projection deformation at the central meridian is 0, and the projection deformation value increases with the distance from the central meridian". The central meridian is selected to be far from the eastern end and close to the western end; there are two possible situations: this meridian is on the route, and it is considered that this meridian is located in the west. Finally, the central meridian of the survey area is selected as 105°15′. After selecting 105°15′ as the central meridian, the trend of Gauss projection length changing from west to east is shown in Figure 5, and it is considered that this meridian is located in the west. Finally, the central meridian of the survey area is selected as 105°15′.

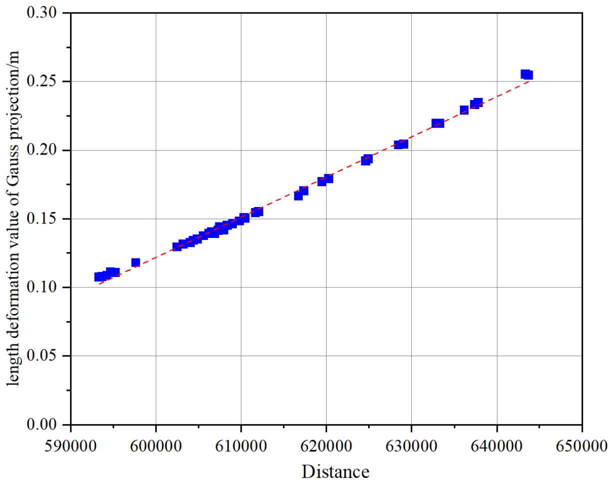

Fig 5 Trend Chart of Gauss Projection Length Deformation Value

It can be seen from the figure above that the trend of length change caused by Gaussian projection deformation is the same as that caused by elevation change, but since their signs are opposite, they can compensate each other, so better results can be achieved. If the projection surface for elevation change is the average elevation surface of the survey area, and the meridian passing through the center of the survey area is used as the central meridian of the survey area, it cannot be guaranteed to meet the specification requirements. The final method selected should be based on a detailed analysis of the terrain and location of the surveyed area. If possible, conduct a detailed analysis of the terrain and location of the surveyed area, and select the most effective method through calculation and comparison.

The average elevation of the entire expressway from Taibai to Fengxian is 1100 meters, which belongs to a high-altitude linear project. When adopting the traditional central meridian scheme, due to the large elevation difference and large horizontal deviation, the deformation value in the western part of the survey area approaches 2.6 cm/km, slightly exceeding the specification limit. In this study, a combined scheme of "optimized central meridian + compensation elevation surface" is adopted, with the central meridian set at 105°15′ (close to the western end of the survey area) and the compensation elevation surface set at 1100 meters (the average elevation of the entire line), so as to realize accurate reverse complementarity between Gaussian projection elongation deformation and elevation reduction short deformation. Through program calculation and measurement, the distribution of the entire regions projection deformation values after optimization is uniform, with the maximum deformation value being 2.4 cm/km and the minimum being 1.8 cm/km, both controlled within the specification limit of 2.5 cm/km. This thoroughly solves the problem of excessive deformation in high-altitude areas, ensures the accuracy of measurement results throughout the entire process of route survey, construction setting-out, and completion acceptance, and provides empirical reference for coordinate system design of similar high-altitude linear projects.

4.4. Adaptation Results of Different Engineering Coordinate System Selections

Based on the research results of engineering cases, this paper summarizes the adaptation schemes for the selection of high-speed railway engineering coordinate systems under different working conditions, forming a standardized selection strategy: ① North-South oriented high-speed railway projects: with large longitudinal span and small transverse span of the line, the scheme of standard central meridian, adaptive compensated elevation surface is preferred, and there is no need to adjust the zone division, so as to achieve precise control of deformation; ② East-West cross-zone high-speed railway projects: the line spans multiple Gauss 3° zones, requiring the establishment of multiple independent coordinate systems by regions, with the sub-zone boundary avoiding large structures such as bridges and tunnels, and establishing precise conversion relations between adjacent coordinate systems to ensure smooth connection; ③ High-altitude undulating high-speed railway projects: adopt the ellipsoid expansion method to construct the compensated coordinate system, and optimize the position of the central meridian simultaneously to realize two-way compensation of elevation deformation and projection deformation; ④ Short-distance gentle high-speed railway projects: directly adopt the national 3° zone standard coordinate system to simplify the process and reduce the operation cost. Verified by actual measurement, all the above schemes can stably control the projection deformation within the specification limit, perfectly adapting to the measurement needs of high-speed railway projects under different working conditions.

5. Conclusions

This paper conducts a systematic research on the selection of coordinate systems and coordinate transformation for high-speed railway engineering. Through theoretical analysis, model construction and case verification, the following conclusions are drawn:

(1) The projection deformation of long and narrow linear engineering for high-speed railways is mainly caused by both elevation reduction and Gauss projection. The unified national coordinate system cannot meet the accuracy requirements, and the custom-made compensated engineering coordinate system adapted to arbitrary zones is an effective way to control deformation. By optimizing the central meridian and compensated elevation surface, the reverse compensation of the two types of deformation can be realized.

(2) Both the ellipsoid expansion method and the ellipsoid translation method can construct a reference ellipsoid adapted to the engineering elevation. Among them, the ellipsoid expansion method is more suitable for high-speed railway engineering applications due to its convenient calculation and strong coordinate compatibility. The Bursa seven-parameter model can achieve accurate transformation between different coordinate systems, and the transformation accuracy meets the requirements of high-speed railway engineering survey.

(3) The verification of engineering examples shows that the optimized engineering coordinate system can control the projection deformation within 1/43000, which is far lower than the specification limit of 2.5 cm/km. The coordinate

system selection strategy and transformation method proposed in this paper have strong practicality and operability, and can provide references for linear engineering such as high-speed railways and expressways.

Reference

[1] Ning X, Zhu Q, Zhang H, et al. Dynamic simulation method of high-speed railway engineering construction processes based on virtual geographic environment[J]. ISPRS International Journal of Geo-Information, 2020, 9(5): 292.

[2] Yong L E I, Lihui W, Zhu C, et al. Exploration on the selection of Engineering Project coordinate system based on" Integration of multiple regulations"[C]//IOP Conference Series: Earth and Environmental Science. IOP Publishing, 2021, 658(1): 012043.

[3] Wu Y, Qian X. The Establishment of Engineering Coordinate System and Its Coordinate Transformation by Changing the Geometric Parameters of Ellipsoid[C]//IOP Conference Series: Earth and Environmental Science. IOP Publishing, 2021, 719(4): 042057.

[4] Bremner M, Santos M. A local projection for integrating geodetic and terrestrial coordinate systems[J]. Survey review, 2020.

[5] Klatte-Schulz F, Thiele K, Scheibel M, et al. Subacromial bursa: a neglected tissue is gaining more and more attention in clinical and experimental research[J]. Cells, 2022, 11(4): 663.

[6] Mutlu Ö, Şahin A. Investigating the effect of modeling approaches on earthquake behavior of historical masonry minarets-Bursa Grand Mosque case study[J]. Sigma, 2016, 7(2): 123-136.

[7] Grossman W. Bearings-only tracking-A hybrid coordinate system approach[J]. Journal of guidance, control, and dynamics, 1994, 17(3): 451-457.

[8] Walpole L J. A translated rigid ellipsoidal inclusion in an elastic medium[J]. Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences, 1991, 434(1892): 571-585.

[9] Wang Y M, Yang X. On the spherical and spheroidal harmonic expansion of the gravitational potential of the topographic masses[J]. Journal of Geodesy, 2013, 87(10): 909-921.