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A Review of Soil Constitutive Model Research: From Classical Theories to Frontier Advances

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Abstract: Soil constitutive models are mathematical expressions that describe the stress-strain-strength-time relationships of soil mass, and are essential for accurately predicting the deformation behavior of soil under external loading. This paper systematically reviews the development history of soil constitutive models both domestically and internationally. Internationally, the Cambridge model proposed by Roscoe et al. in 1963 marked the beginning of the modern stage of soil mechanics. Domestically, the concept of constitutive models began to appear in the 1970s, and with the development of computer technology and the finite element method, research on such models has gradually gained practical significance. On this basis, this paper introduces various types of constitutive model advances, including the BBM model for unsaturated soils, and typical recent achievements: a mechanical model for frost heave of trapezoidal canals based on the two-parameter elastic foundation beam theory, a three-dimensional modified Cam-clay model for structured soils considering intermediate principal stress, a modified Yin Zongze model for biopolymer-improved expansive soil, and a nonlinear dynamic constitutive model for soils considering fluid-solid coupling effects and its implementation in FLAC3D. Finally, future directions for constitutive model development are discussed, emphasizing the integration of physical mechanisms, mathematical methods, and computer technology to improve model accuracy and applicability in practical engineering. This paper serves as a theoretical reference for research in soil mechanics.

Keywords: Soil constitutive model, Cambridge model, elastoplastic model, unsaturated soil, structured soil, fluid-solid coupling, nonlinear dynamic constitutive model.

1. Introduction

A constitutive model, also referred to as the mechanical constitutive equation of a material or the stress-strain model of a material, is a mathematical expression describing the mechanical behavior (stress-strain-strength-time relationship) of a material. The stress-strain relationship of materials is highly complex, exhibiting nonlinearity, viscoelastic-plasticity, dilatancy, anisotropy, etc. Meanwhile, factors such as stress level, stress history, material composition state, and structure all influence this relationship. Soil, as a naturally complex porous material, undergoes deformations under external loading characterized by nonlinearity, rheology, anisotropy, dilatancy, and other features. To better describe the true mechanical deformation behavior of soil, it is necessary to establish the relationships among stress, strain, strength, and time. Consequently, based on various experiments and engineering practice experiences, a mathematical model—namely the soil constitutive model—is proposed. Since Roscoe et al. developed the Cambridge model between 1958 and 1963, scholars worldwide have successively proposed various soil constitutive models oriented toward different research directions, including time-independent linear elastic models, nonlinear elastic models, elastoplastic models, and time-dependent rheological models. In China, the term "constitutive model" began appearing in soil mechanics only in the 1970s, and at that time it was also referred to as the stress-strain mathematical model. This paper will combine the research progress of soil constitutive models and introduce some innovations based on existing models as learned from the literature.

2. Development History of Constitutive Models at Home and Abroad

(1) Research on Constitutive Models Abroad

Roscoe et al. from Cambridge University, UK, proposed a model in 1963. Based on drained and undrained triaxial tests on normally consolidated and overconsolidated soil specimens, the model introduced the concept of the critical state of soil. On the basis of experiments, it incorporated the principles of work hardening and energy equations. This model theoretically and experimentally elucidated the elastoplastic deformation characteristics of soil, particularly considering the plastic volumetric deformation of soil. Therefore, it is generally recognized that the advent of the Cambridge model marked the entry of soil constitutive theory development into the modern stage of soil mechanics.

(2) Research on Constitutive Models in China

In the 1970s, the term "constitutive model" began to appear in the field of soil mechanics in China. The increasing attention to constitutive model research in soil mechanics has its historical background and reasons. As a technical discipline, soil mechanics has a long history internationally, and it has been an independent discipline in China for nearly 80 years. However, our understanding of the engineering characteristics of soils was still very superficial at that time. Professor Huang Wenxi mentioned this point in a report in Shanghai in 1978, stating: "In terms of computational theory, the stability analysis of soil masses and earth pressure calculations employ classical plasticity theory established for perfectly plastic materials, while settlement analysis of soil masses uses classical elasticity theory established for perfectly elastic materials. Such a high degree of simplification is unsatisfactory. The main reason why this situation persisted for many years was primarily the limitations of computer technology." The "limitations" mentioned by Professor Huang Wenxi at that time now seem to be resolvable. With the gradual development of computers in China in recent years, complex computational problems have become much easier to solve. Numerical analysis methods applied in soil mechanics, primarily the finite element method, have given practical significance to the study of soil constitutive models.

3. Types of Constitutive Model Development

The Cambridge model was established using the critical state elastoplastic model for saturated soils, providing a good approach for research on saturated soils. However, how to apply the saturated soil critical state elastoplastic model to unsaturated soils has long puzzled researchers in unsaturated soil mechanics. Consequently, Alonso et al. proposed the BBM model (Alonso model) for unsaturated soils in 1990. Under the influence of this model, research on elastoplastic constitutive models for unsaturated soils became a hotspot in soil mechanics. The BBM model is a typical elastoplastic model based on net stress and matric suction. It can describe many mechanical behaviors of unsaturated soils, such as yield stress increasing with suction and collapse deformation upon wetting. This model provides a complete theoretical framework to describe the different properties and characteristics of unsaturated soils, laying the theoretical foundation for constitutive models describing more complex phenomena.

Many constitutive models have been established, which can be broadly classified into the following categories: elastic models for soils, hyperelastic models for soils, hypoelastic models for soils, viscoelastic models for soils, elastoplastic models for soils, viscoelastoplastic models for soils, and endochronic plastic models for soils. Most soil constitutive models are built upon the aforementioned existing models. The following are some recent constitutive models I have learned from the literature.

In the Transactions of the Chinese Society of Agricultural Engineering, Professor Xiao Min and his research group published a paper titled [1] "A Mechanical Model for Frost Heave of Trapezoidal Canals Based on the Two-Parameter Elastic Foundation Beam Theory," in which they established a constitutive model. This model was developed to overcome the shortcomings of existing trapezoidal canal elastic foundation beam models, such as neglecting soil continuity and requiring a presupposed distribution of tangential frost heave forces. Based on the Winkler model, the study uses the extension and contraction of soil springs to describe the normal frost heave force and normal adfreeze force, and introduces a shear layer and a contact interface layer to construct a two-parameter frozen soil foundation beam model for trapezoidal canals. The shear layer accounts for interactions between soil springs, and the contact interface layer integrates the calculation of tangential adfreeze force into the model for unified solution. This model can provide a reference for the design of frost heave resistance in trapezoidal canals.

In the Chinese Journal of Geotechnical Engineering, Professor Li Hangzhou's paper [2] "A Three-Dimensional Modified Cam-clay Model for Structured Soils" extends the Cambridge model to develop a constitutive model for structured soils. Recognizing that soil structure and stress state are important factors influencing soil mechanical behavior, the model works within the framework of critical state soil mechanics. It considers the effect of the intermediate principal stress, incorporates the unified strength theory, introduces the Lode angle to derive unified strength parameters, and modifies the critical state line to reflect the critical state under different stress states. Considering the influence of soil structure, it is introduced into the yield function of the modified Cam-clay model. Using a non-associated flow rule, the dilatancy stress equation is determined, and a structured constitutive model reflecting the influence of the intermediate principal stress is established. The proposed constitutive equation is validated using true triaxial tests on cohesive soil. The results show that the established constitutive model can reasonably predict the constitutive behavior under three-dimensional stress states, confirming the model's rationality.

In Rock and Soil Mechanics, Professor Chen Yongqing and his research group published [3] "A Modified Yin Zongze Model for Biopolymer-Improved Expansive Soil." This paper aims to describe the stress-strain relationship of biopolymer-improved expansive soil. It proposes modifying the model established by Professor Yin Zongze by incorporating a biopolymer into the expansive soil. For this purpose, a series of drained triaxial shear tests were conducted. The elastic deformation behavior of expansive soil improved with different biopolymer contents was analyzed, and the variation patterns of the model's shear compression and shear dilation related parameters with biopolymer content were studied. The influence of biopolymer content on the parameters of the Yin Zongze model was analyzed, and the biopolymer content was introduced as a modification factor to amend the Yin Zongze model. Experimental results indicate that the biopolymer effectively enhances the mechanical properties of expansive soil. The modified Yin Zongze model can better describe the constitutive relationship of biopolymer-improved expansive soil. A comparison between theoretical calculations using the modified Yin Zongze model and experimental values shows high agreement. Moreover, the parameter determination method for the modified Yin Zongze model is the same as that for the original Yin Zongze model.

In the Chinese Journal of Geotechnical Engineering, the Institute of Geotechnical Engineering at Nanjing Tech University published a paper titled [4] "A Nonlinear Dynamic Constitutive Model for Soils Considering Fluid-Solid Coupling Effects and Its Three-Dimensional Numerical Implementation." The paper presents a nonlinear dynamic constitutive model for soils that can describe fluid-solid coupling effects. This model arises from the complex fluid-solid coupling effect induced by wave action in the seabed, where the soil experiences three-dimensional coupled shear from normal stress differences and shear stresses, transient excess pore water pressure exhibits both oscillatory and accumulative growth modes, and the soil undergoes cyclic softening. Existing Masing-type nonlinear dynamic constitutive models do not fully account for this fluid-solid coupling effect. The study extends the Davidenkov skeleton curve to three-dimensional stress space, uses the "extended Masing" rule to construct soil stress-strain hysteresis curves, and introduces the shear-volume strain coupling model proposed by Byrne into the volumetric compatibility equation as a source term for cumulative excess pore pressure growth based on Biot's dynamic consolidation theory. A nonlinear dynamic constitutive model for soils capable of describing fluid-solid coupling effects is thus established. The model is secondarily developed on the FLAC3D computational platform. Agreement with theoretical model values ensures computational accuracy, and comparisons with cyclic torsional shear tests on soil elements verify the feasibility and rationality of the constitutive model for simulating dynamic fluid-solid coupling effects in soils, providing an effective means for marine geotechnical engineering research using FLAC3D.

4. Future Prospects for Constitutive Models

The core challenge in establishing soil constitutive models is to characterize the intrinsic constitutive relationship through the physical properties of soil observed in experiments. In recent years, with the rapid development of computer technology, a large number of nonlinear theories and methods have emerged, making it possible to construct more complex constitutive relationships through numerical computations within the models themselves. However, the accuracy and applicability of these models remain key issues worthy of in-depth discussion. Currently, many constitutive models perform well under specific experimental conditions but often exhibit significant deviations when extended to more complex stress paths, drainage conditions, or soil types. Therefore, in future research, we need to pay close attention to the discrepancies between the assumptions of soil constitutive models and actual engineering conditions. Many models are based on idealized assumptions such as homogeneity, continuity, and isotropy, whereas natural soils often exhibit characteristics such as structure, anisotropy, unsaturation, and spatial variability. Directly applying these models can significantly affect the accuracy and reliability of engineering predictions. Subsequent research should focus on improving and enhancing existing models, shifting from empirical fitting toward more physically grounded mechanisms, striving to make models more aligned with practical engineering needs and better serve engineering practice. Specifically, future development directions can be pursued from the following aspects:

(1) Multi-scale constitutive modeling and micro-macro linkage mechanism research

The macroscopic mechanical behavior of soils originates essentially from particle-scale interactions, rearrangements, and breakage. Traditional constitutive models are mostly based on the continuum mechanics framework and struggle to capture the influence of microstructural features such as particle morphology, gradation, cementation, and pore structure. Future research should strengthen the investigation of the linkage mechanisms between soil microstructure (e.g., particle orientation, pore distribution, contact force chains) and macroscopic mechanical responses. By employing high-precision microscopic observations (e.g., X-ray CT, scanning electron microscopy) and discrete element numerical simulations, a macroscopic constitutive framework that reflects micro-evolutionary laws should be established. For example, particle-scale mechanisms of energy dissipation, dilatancy, and breakage can be introduced into continuum models through homogenization theory or internal variable methods, leading to the development of genuinely "micromechanics-inspired constitutive models."

(2) Data-driven and physics-informed constitutive models

With the rapid advancement of artificial intelligence and machine learning, purely data-driven constitutive modeling methods (e.g., neural networks, support vector machines, Gaussian process regression) have demonstrated powerful capabilities in fitting complex stress-strain relationships. However, pure black-box models often lack physical interpretability and have limited extrapolation ability. An important future trend is to embed physical laws (e.g., thermodynamic conservation, irreversible thermodynamics framework, yield surface consistency conditions) into machine learning models to develop "physics-informed" constitutive models. For example, physics-informed neural networks (PINNs) or symbolic regression methods can be used to automatically discover constitutive equations that satisfy thermodynamic constraints from experimental data. This approach ensures theoretical rigor while fully leveraging the flexibility of data-driven methods.

(3) Constitutive theories considering multi-field coupling and environmental effects

In practical engineering, soils are often subjected to multi-field coupling involving temperature fields (frozen soils, high-temperature environments), chemical fields (contaminants, saline water, biopolymer improvement), and seepage fields (unsaturated seepage, excess pore pressure). Most existing models only consider mechanical-seepage coupling, while the description of thermo-hydro-mechanical-chemical (THMC) multi-field coupling effects remains incomplete. Future efforts should focus on developing constitutive models that systematically account for the influences of temperature, humidity, chemical concentration, and biological actions, particularly for special conditions such as cold-region frozen soils, landfill covers, contaminated site remediation, and biopolymer-improved soils. At the same time, further research on parameter calibration and numerical implementation of coupled models is needed to lower the threshold for application.

(4) Improvement and standardization of constitutive models for unsaturated and special soils

Although unsaturated soil elastoplastic models represented by the BBM model have made significant progress, they still have shortcomings in describing drying-wetting cycle hysteresis effects, suction history dependence, and water retention hysteresis characteristics. For special soils such as expansive soils, collapsible loess, and soft soils, existing modified models (e.g., the Yin Zongze model and Cam-clay extensions mentioned in this paper) are mostly targeted at single improvement methods or specific stress paths, lacking a unified theoretical framework. Future research should combine advanced testing equipment (e.g., unsaturated soil triaxial apparatuses, suction-controlled tests) to obtain more comprehensive data, and on this basis develop models capable of uniformly describing saturated-unsaturated states, wetting-drying cycles, and structural damage-healing processes.

(5) Parameter identification and uncertainty quantification for constitutive models

Existing constitutive models often involve numerous parameters with correlations among them, and parameters measured in the laboratory frequently differ significantly from those back-analyzed from field data. Future research should strengthen the application of uncertainty quantification methods such as Bayesian inversion and Markov chain Monte Carlo (MCMC) to establish parameter confidence intervals and model prediction reliability assessment systems. At the same time, developing simple, rapid, and non-destructive parameter acquisition techniques (e.g., based on laboratory bender elements, CPTU in-situ tests) is also an important direction for promoting the engineering application of constitutive models.

In summary, future research on constitutive models is no longer a single-disciplinary endeavor but requires deep interdisciplinary integration of soil mechanics, solid mechanics, computational mathematics, materials science, and artificial intelligence. Only through extensive engineering practice and physical experimentation, combined with collaborative innovation in theoretical construction, numerical methods, parameter identification, and uncertainty analysis, can soil constitutive models truly serve complex engineering problems and propel modern soil mechanics toward more accurate, reliable, and intelligent development.

5. Conclusion

This paper systematically reviews the development history, main types, and recent research advances in soil constitutive models, and based on this analysis, provides an outlook on future directions. The following main conclusions can be drawn:

(1) Soil constitutive models have evolved from early linear elastic models and the classical Cam-clay model to complex models covering unsaturated soils, structured soils, improved soils, and fluid-solid coupling effects. This evolution has progressed from empirical fitting to theory-experiment integration, and from single stress states to multi-dimensional stress paths. The Cam-clay model laid the theoretical foundation for modern soil mechanics, while the BBM model successfully extended the elastoplastic framework to unsaturated soils, serving as a cornerstone for subsequent research.

(2) In recent years, researchers worldwide have made numerous innovative extensions based on traditional models. For example, the canal frost heave model based on the two-parameter elastic foundation beam theory introduces a shear layer and a contact interface layer, overcoming the shortcomings of disregarding soil continuity and presupposing tangential force distribution in trapezoidal canal frost heave analysis. The three-dimensional modified Cam-clay model incorporates the unified strength theory and the Lode angle, achieving a reasonable description of the intermediate principal stress effect in structured soils. The modified Yin Zongze model introduces a biopolymer content factor, significantly improving the prediction accuracy of the stress-strain relationship of improved expansive soil. The nonlinear dynamic constitutive model considering fluid-solid coupling effects extends the Davidenkov skeleton curve to three-dimensional stress space and combines it with Biot's consolidation theory, achieving effective simulation of marine geotechnical engineering through secondary development in FLAC3D. These results indicate that modifying classical model frameworks by incorporating specific physical mechanisms or material parameters remains one of the mainstream approaches in current constitutive model development.

(3) Although existing models perform well under specific conditions, they commonly suffer from simplified assumptions, complex parameter determination, and reduced accuracy when extended to practical engineering. Future development of constitutive models should focus on the following directions: multi-scale micro-macro linkage mechanisms, physics-informed data-driven modeling, multi-field coupling effects such as thermo-hydro-mechanical-chemical (THMC) interactions, unification of unsaturated and special soil models, parameter identification and uncertainty quantification, and deep integration with high-performance computing and digital twin technologies.

(4) Overall, research on soil constitutive models has entered a new stage characterized by interdisciplinary integration, multi-field coupling, and the parallel advancement of data-driven methods and traditional theories. Only by continuously strengthening fundamental experiments, deepening the understanding of physical mechanisms, and combining modern mathematical, mechanical, and artificial intelligence approaches can we construct advanced constitutive models that better meet practical engineering needs, thereby providing a solid theoretical foundation for the safety and reliability of geotechnical engineering design..

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