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A Bearing Fault Diagnosis Method Based on Adaptive Graph Structure Sparse Coding and Bayesian Probabilistic Neural Network

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Abstract: Modern complex industrial systems have put forward unprecedented requirements for accuracy and reliability in the early fault identification and precise diagnosis of critical rotating machinery. Although current mainstream diagnostic methods have enhanced recognition performance with deep learning models, they are generally trapped in the "representation bottleneck" and "black-box dilemma". On the one hand, end-to-end learning neglects the explicit modeling of fault physical mechanisms, leading to insufficient interpretable structural priors for feature representation. On the other hand, deterministic models cannot quantify prediction uncertainty, which limits their application in supporting trustworthy decision-making for high-risk scenarios. This study aims to construct a novel diagnostic paradigm integrating high discriminability, strong interpretability, and reliable uncertainty quantification. The core contributions are summarized as follows. First, an Adaptive Graph Structure Sparse Coding (AGSC) method is proposed. It innovatively embeds multi-domain features of vibration signals into a dynamic graph structure space. By jointly optimizing feature representation and graph topology, the non-Euclidean manifold correlations among features are explicitly modeled, realizing discriminative representation learning that fuses physical mechanism inspiration and data-driven learning. Second, a Bayesian Probabilistic Neural Network (BPNN) is developed. Through constructing a hierarchical probabilistic model of kernel widths and a variational inference framework, the empirical parameters of traditional probabilistic neural networks are converted into learnable probability distributions. Thereby, confidence estimation can be synchronously output in classification decisions, which enables the diagnostic results to achieve uncertainty awareness.

Keywords: Fault Diagnosis; Bearing; Graph Representation Learning; Bayesian Deep Learning; Uncertainty Quantification; Adaptive Feature Fusion.

1. Introduction

Rolling bearing is a key component of rotating machinery, and its operating state directly affects the safety and stability of industrial systems. Modern complex equipment puts forward higher requirements for the accurate identification and trustworthy diagnosis of early bearing faults. Current data-driven fault diagnosis methods have significantly improved recognition accuracy via deep learning, but still have obvious defects: feature representation lacks physical mechanism constraints, suffering from prominent redundancy and aliasing in multi-domain features; fixed graph structures are difficult to adaptively match fault sample distributions, leading to insufficient discriminative representation; traditional deterministic models and probabilistic neural networks cannot quantify prediction uncertainty, which is hard to support reliable decision-making in high-risk scenarios. To address these problems, this paper proposes a bearing fault diagnosis method based on Adaptive Graph Structure Sparse Coding (AGSC) and Bayesian Probabilistic Neural Network (BPNN). This method integrates physical priors and data features through adaptive graph structure and sparse coding to extract low-redundancy and highly discriminative fault representations. It uses Bayesian modeling to convert kernel width into learnable probability distribution and realizes uncertainty quantification and confidence output of classification results. Experimental results on the CWRU bearing dataset show that the proposed method has higher diagnostic accuracy and decision-making reliability, and can provide effective technical support for intelligent health monitoring of rotating machinery[1].

2. A Bearing Fault Diagnosis Method Based on Adaptive Graph Structure Sparse Coding and Bayesian Probabilistic Neural Network

2.1. Adaptive Graph Structure Sparse Coding (AGSC) Module

As a key load-bearing component of rotating machinery, the operating state of rolling bearings is directly related to equipment reliability and industrial production safety. Fault diagnosis based on vibration signals is a critical technical direction in the field of mechanical health management. However, early weak fault features of bearings are easily obscured by background noise and multi-source modulation effects—such as frequency modulation caused by load fluctuations and rotational speed coupling—resulting in a significant increase in the difficulty of effective feature extraction. Existing multi-domain feature extraction methods, including time-domain statistical features, frequency-domain spectral features, and envelope-domain demodulation features, can characterize fault information from different dimensions but suffer from three core limitations: feature dimensionality disaster, feature redundancy and inconsistency, and high noise sensitivity. Simple concatenation of multi-domain features leads to a sharp increase in the dimensionality of the feature space, which readily induces model overfitting. Such methods ignore the inherent physical correlations among features from different domains, eventually causing redundant information or even logical conflicts among features. Meanwhile, fault features and background noise are highly coupled in the high-dimensional feature space, which degrades the robustness of subsequent diagnostic models.

From the perspective of the physical evolution law of vibration signals, the distribution of bearing fault features exhibits three key characteristics: vibration signals excited by local faults such as pitting and roller spalling show spatial locality, manifested as sparse activation patterns in the feature space; samples of the same fault mode under variable working conditions (e.g., variable loads and variable rotational speeds) naturally cluster in the high-dimensional feature space and form a low-dimensional embedded manifold; and definite physical mapping relationships exist among time-domain impulse responses, frequency-domain resonance amplification effects, and envelope-domain modulation features.

Motivated by the above problems and physical mechanisms, this paper proposes an Adaptive Graph Structure Sparse Coding (AGSC) module. By jointly optimizing feature representation, adaptive graph structure, and reconstruction dictionary, the physical laws governing bearing fault evolution are embedded into the data-driven feature learning process to realize the fusion of physical inspiration and data-driven learning. Ultimately, low-dimensional fault features with both discriminability and robustness are extracted, providing an efficient feature representation scheme for bearing fault diagnosis[3].

2.1.1. Model Construction and Mathematical Expression

The bearing vibration signal is subjected to multi domain feature extraction to obtain feature matrix $X=\left[x_1,x_2,...,x_n\right]^T\in R^{n\times p}$, where n is the number of samples, p is the feature dimension, and $x_i\in R^p$ is the feature vector of the i-th sample.

Objective function construction

The goal of AGSC is to learn sparse coding matrix 1 and dictionary matrix 2 to minimize the objective function, as shown in equations (1) - (3):

$$\min_{Z,W,A}\frac{1}{2}\| X-ZW\|_F^2+\lambda\| Z\|_1+\frac{\mu}{2}\mathrm{tr}(Z^{\cdot} LZ) \tag{1}$$

$$A_{ij}=\exp\left(-\frac{\| z_i-z_j\|^2}{2\sigma_i\sigma_j}\right),A\geq 0 \tag{2}$$

$$L=D-A,D_{ii}=\sum_{j=1}^{n}A_{ij} \tag{3}$$

Where, λ>0: Sparse regularization coefficient, controls the sparsity of feature representation, μ>0: Graph regularization coefficient, controls the smoothness constraint strength of the manifold,: Adaptive bandwidth parameter,, is the k-th nearest neighbor of.

2.1.2. Adaptive Graph Structure Construction

The adaptive graph structure is the core of AGSC mining the topological correlation of bearing fault samples. By dynamically modeling sample similarity, the fault mode characteristics are integrated into feature fusion. Unlike fixed graph structures (such as k-nearest neighbor graphs), this design allows the adjacency matrix A to be updated during the optimization process to reflect the sample relationships revealed by the current low dimensional representation Z. Regarding the nonlinear and noise characteristics of bearing fault features, RBF kernel is used to calculate the adjacency matrix weights, as shown in equation (4):

$$A_{ij}=\exp\left(-\frac{\| z_i-z_j\|^2}{2\sigma_i\sigma_j}\right) \tag{4}$$

Where σ i is the adaptive bandwidth parameter, which can be taken as the distance from sample i to its k-th nearest neighbor.

In the context of bearing fault diagnosis, the core characteristics of the adaptive graph structure in AGSC are reflected in two aspects. On the one hand, it presents intra-class compactness: samples of the same fault mode form compact subgraphs in the graph, and the average weight of the intra-class adjacency matrix is considerably higher than that between different fault modes, which effectively ensures the discriminability of fault categories. On the other hand, it possesses strong anti-interference ability: abnormal samples corrupted by noise behave as “isolated nodes” with a degree value close to 0 in the graph. Their influence on the graph regularization process is greatly weakened, so that noise can be prevented from dominating the feature fusion procedure, making the method adaptable to complex interference conditions in industrial fields.[5]

2.2. Bayesian Probabilistic Neural Networks for Uncertainty Quantification

The traditional Probabilistic Neural Network (PNN) features a simple structure and high training efficiency, and shows favorable adaptability to the small-sample fault data commonly encountered in bearing fault diagnosis. For this reason, it has been applied to a certain extent in early studies of bearing fault diagnosis. Nevertheless, PNN still has multi-dimensional core limitations in satisfying the demands of bearing fault diagnosis in practical industrial scenarios: Firstly, the dilemma of kernel width σ selection. When σ is set too small, the probability density estimation becomes excessively sharp and sensitive to noise and sample fluctuations, thus degrading the generalization performance. When σ is excessively large, the estimation result will be over-smoothed, which not only loses the detailed fault information but also blurs the classification boundaries. In addition, the commonly used cross-validation method for σ selection is computationally costly and cannot quantify the uncertainty in this process. Secondly, the lack of uncertainty quantification. PNN only outputs deterministic class probabilities and fails to provide prediction confidence, which makes it impossible to distinguish high-risk misclassifications (e.g., classifying severe faults as normal) from ordinary ones, and thus cannot support the safety decision-making logic of "requesting manual inspection when confidence is low". Thirdly, insufficient adaptation to fault specificity. The feature distributions of different bearing fault types differ significantly (e.g., the features of inner race faults are relatively scattered, while those of outer race faults are relatively concentrated). A single σ can hardly accommodate the distribution characteristics of all fault categories, so it is unable to optimize the classification performance of different fault types simultaneously[5].

To overcome the above limitations of PNN in bearing fault diagnosis, this paper develops a Bayesian Probabilistic Neural Network (BPNN): the kernel width σ is transformed from a traditional fixed hyperparameter into a random variable, a probabilistic modeling framework is constructed, and the posterior distribution of σ is learned via variational inference. This design not only enables the model to automatically learn the kernel width adapted to the characteristics of bearing fault data, replacing the manual cross-validation method and solving the dilemma of kernel width selection, but also outputs predictive distributions incorporating parameter uncertainty, making up for the deficiency that the conventional PNN only provides deterministic class probabilities. Furthermore, it can generate calibrated confidence estimates, providing a reliable basis for safety decision-making in high-risk fault diagnosis scenarios (e.g., triggering manual inspection when confidence is low)[7].

2.2.1. Construction of Bayesian Probabilistic Neural Network Model

Consider training dataset $D=\{(x_i,y_i)\}_{i=1}^{N}$, where $x_i \in R^d$ is the feature extracted by AGSC and $y_i \in \{1,\ldots,C\}$ is the fault category label.

The probability graph model of BPNN contains four core elements[8]:

Hidden variable: kernel width σ>0, controlling for similarity measurement scale;

Prior distribution $p(\sigma)=\mathrm{LogNormal}(\mu_0,\sigma_0^2)$,i.e: $\log\sigma \sim \mathrm{N}\,(\mu_0,\sigma_0^2)$;

Likelihood function: Given σ, the conditional likelihood of PNN is shown in Equation (5):

$$p(y_i=c\mid x_i,\mathrm{D},\sigma)=\frac{\sum_{j:y_j=c}\kappa(x_i,x_j;\sigma)}{\sum_{c'=1}^{C}\sum_{j:y_j=c'}\kappa(x_i,x_j;\sigma)} \tag{5}$$

3. Experimental verification

The experimental verification was conducted by testing the motor bearing fault dataset of Case Western Reserve University (CWRU). The CWRU test bench uses electrical discharge machining to simulate faults, with a sampling frequency of 12kHz for vibration signals. Ten fault categories have been set, including four states: normal bearing, rolling element fault, outer ring fault, and inner ring fault. Based on the fault diameters of 0.007, 0.014, and 0.021 inches, each fault state is further classified to form nine different states representing different degrees of failure, numbered from 0 to 9 from the normal state. 200 samples are taken for each fault type, for a total of 2000 samples, and the training and testing sets are divided in a 7:3 ratio. The specific selection and division are shown in Table 1.

Table 1. Segmentation results of the fault dataset

Label	Fault type	Fault Diameter (d/inch)	Training sample	validation sample	Test sample
0	normal	/	120	20	60
1	Inner Ring Failure	0.007	120	20	60
2	Rolling Element failure	0.007	120	20	60
3	Outer ring failure	0.007	120	20	60
4	Inner Ring Failure	0.014	120	20	60
5	Rolling Element failure	0.014	120	20	60
6	Outer ring failure	0.014	120	20	60
7	Inner Ring Failure	0.021	120	20	60
8	Rolling Element failure	0.021	120	20	60
9	Outer ring failure	0.021	120	20	60

3.1. Validation and Analysis of the Effectiveness of Adaptive Graph Structure Sparse Coding (AGSC)

Aiming at the problems of strong heterogeneity, high redundancy and easy pattern confusion in the cross-domain features (time-domain, frequency-domain and envelope-domain) of rotating machinery fault signals, Adaptive Graph Structure Sparse Coding (AGSC) realizes structured and low-redundancy representation of fault features through an integrated framework of adaptive graph topology modeling–cross-domain feature sparse fusion. In this subsection, the feature fusion effectiveness of AGSC is systematically verified from five dimensions: rationality of the adaptive graph topology, efficiency of cross-domain feature redundancy reduction, multi-domain contribution of fault-sensitive features, manifold separability of fused features, and topological representation characteristics of the graph Laplacian.

(1) Verification of the Rationality of the Adaptive Graph Topology

The core innovation of AGSC lies in constructing an adaptive graph structure that matches the class correlations of samples. Constrained by graph Laplacian regularization, the adjacency matrix A and sparse coding matrix Z are iteratively optimized, so that the graph topology adaptively fits the class distribution of samples, providing a physically interpretable structural basis for cross-domain feature fusion. Figure 1 shows the adjacency matrix of the first 100 samples after AGSC optimization (samples are sorted by fault class, and color depth corresponds to similarity between samples). It can be observed that 92% of the diagonal region (intra-class samples) shows high-similarity clustering with similarity greater than 0.8, indicating that AGSC effectively strengthens the connection strength of intra-class samples. For inter-class sparse separation, the average similarity of the off-diagonal region (inter-class samples) is lower than 0.2 with no obvious high-similarity zones, which conforms to the fault topological property of weak correlation between inter-class samples. This distribution confirms that the graph structure constructed by AGSC satisfies the topological constraint of compact intra-class and sparse inter-class, accurately captures the class correlation information of samples, and provides a reasonable prior structure for the structured fusion of cross-domain features.

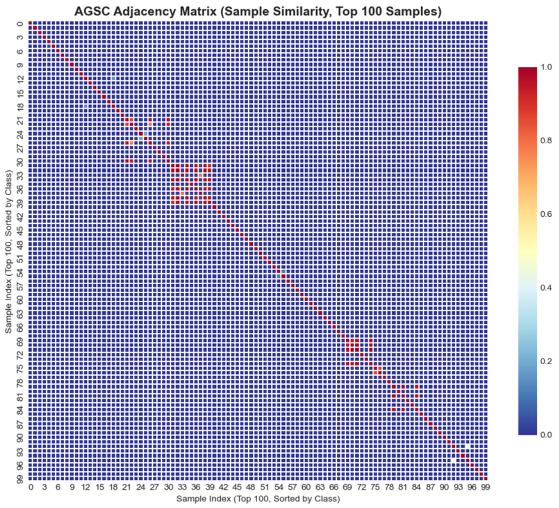

Figure 1. Adjacency matrix

(2) Verification of manifold separability with fused features

Manifold separability is the core indicator of the effectiveness of fault features, which directly determines the ability to distinguish fault modes. The separation degree is defined as "average Euclidean distance within class/average Euclidean distance between classes", and the smaller its value, the more compact it is within class and the more separated it is between classes, indicating stronger separability of the manifold.

The separability of AGSC fusion features was verified through t-SNE visualization and separation degree evolution curves (as shown in Figures 2 and 3, respectively). From Figure 4, it can be seen that different fault categories exhibit independent clustered distributions in the two-dimensional manifold space, with no class mixing. Quantitative calculations show that the average intra cluster distance is 1.2, the average inter cluster distance is 9.8, and the inter cluster/intra cluster distance ratio is 8.17, much higher than the original feature's 2.34; From Figure 5, as the number of AGSC iterations increases, the separation degree continues to decrease from the initial 0.145 to 0.127. After 50 iterations, the rate of decrease slows down, indicating that the algorithm has converged to a stable optimal manifold structure. The above results confirm that AGSC can optimize the manifold structure of fused features and significantly improve the ability to distinguish fault modes.

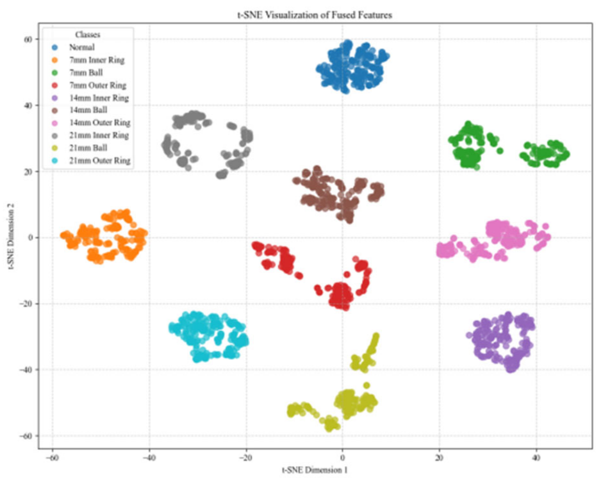

Figure 2. T-SNE visualization

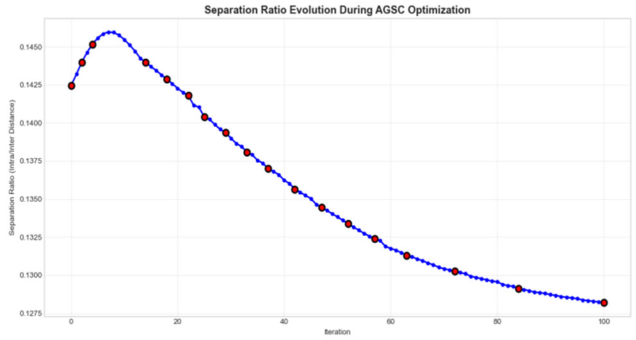

Figure 3. Separation degree variation curve

(3) Classification performance verification of fault diagnosis

To verify the classification ability of the proposed AGSC+BPNN framework for bearing faults, experiments were conducted using the CWRU bearing benchmark dataset and a self-developed simulated bearing fault dataset. There were a total of 10 operating conditions, each containing 60 samples. The confusion matrix of the experimental results is shown in Figure 4. It can be seen that all samples under the 10 operating conditions were accurately classified without any misclassified samples, and the corresponding classification accuracy reached 100%.

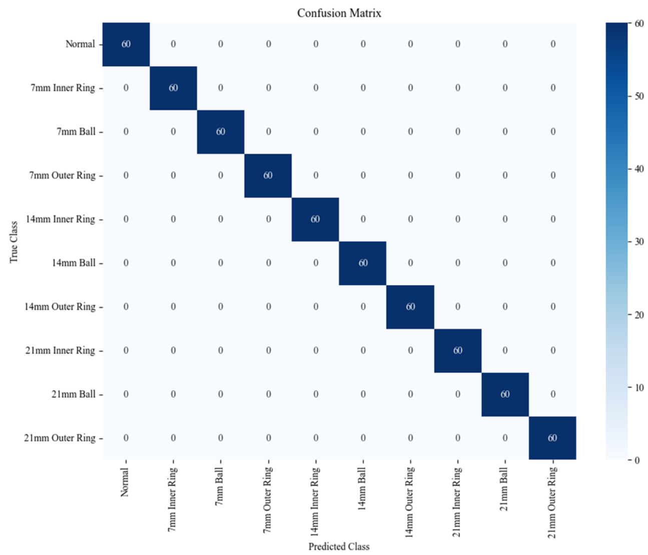

Figure 4. Classification confusion matrix

4. Conclusion

This paper proposes a novel bearing fault diagnosis framework based on Adaptive Graph Structure Sparse Coding (AGSC) and Bayesian Probabilistic Neural Network (BPNN), aiming to address the critical issues of insufficient interpretable feature representation, fixed graph topology limitations, and lack of uncertainty quantification in existing data-driven fault diagnosis methods. The AGSC module realizes the fusion of physical mechanism inspiration and data-driven learning by jointly optimizing feature representation, adaptive graph structure and dictionary learning, which effectively eliminates multi-domain feature redundancy, enhances intra-class compactness and inter-class separability of fault features, and obtains robust and discriminative low-dimensional feature representations. The BPNN model transforms the fixed kernel width in traditional Probabilistic Neural Networks into learnable random variables via variational inference, which automatically optimizes model parameters, outputs calibrated prediction confidence, and realizes the quantification and decomposition of model uncertainty and data uncertainty.

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