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A Unified Battery Discharge Prediction Framework Based on Electro-Thermal Coupling Model and Markov-Modulated Monte Carlo Algorithm

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Abstract: This paper addresses the modeling of dynamic characteristics during the discharge process of smart device batteries by constructing a continuous-time equivalent model that integrates electrical and thermal coupling mechanisms. At the electrical level, an improved first-order Thevenin equivalent circuit is introduced to uniformly model open-circuit voltage, ohmic internal resistance, and polarization effects. Combined with a constant-power load constraint, this model characterizes the nonlinear feedback behavior where voltage drop triggers current amplification; At the thermal level, a temperature evolution model is established based on the equilibrium between Joule heating and convective cooling, while a temperature-dependent internal resistance expression based on the Arrhenius equation is introduced to achieve a bidirectional electro-thermal coupling description. On this basis, the system is transformed into a state-space form to uniformly characterize the cooperative evolution of the charge state, polarization voltage, and temperature. Furthermore, to address random load fluctuations and performance degradation in real-world usage scenarios, a Monte Carlo simulation framework based on continuous-time Markov processes is proposed. This framework incorporates Gaussian perturbations to model load uncertainty and introduces a health state parameter to unify the modeling of capacity degradation and internal resistance increase. Simulation results demonstrate that this method effectively captures discharge behavior and lifespan trends under multi-factor coupling, exhibiting good generalization capability and predictive stability. It provides a unified modeling foundation for battery performance evaluation and energy management.

Keywords: Electro-Thermal Coupling Model; Markov-Modulated Monte Carlo Method; Constant Power Load Model.

1. Introduction

As the performance of smart devices continues to improve, their operation has become increasingly reliant on battery power. Consequently, accurately characterizing battery discharge behavior under complex operating conditions has become a critical issue. Traditional methods are often based on coulomb counting or simplified equivalent circuit models, treating the battery as an ideal energy storage unit and ignoring the coupling relationships among load variations, temperature effects, and internal nonlinear mechanisms. Although these methods are computationally straightforward, they often struggle to explain phenomena such as voltage sags and rapid capacity decay under high-load, low-temperature, or aged conditions, leading to discrepancies between predicted results and actual usage scenarios.

In recent years, research has gradually shifted from modeling single physical processes to multi-mechanism co-modeling, attempting to provide a unified description of the system across multiple dimensions, including electrical and thermal aspects. However, existing models are mostly focused on steady-state or weakly coupled scenarios, with insufficient consideration of feedback effects caused by constant-power loads and the dynamic influence of temperature on internal resistance and capacity. Additionally, there are still limitations in the comprehensive modeling of random load fluctuations and performance degradation. Furthermore, most methods lack a unified algorithmic framework, making it difficult to balance model expressiveness with computational feasibility [1][2].

To address these issues, this paper adopts a continuous-time modeling perspective to construct an electro-thermal coupled equivalent circuit model. It integrates charge state, polarization effects, and temperature evolution into a unified state-space description and introduces a constant-power constraint to characterize the nonlinear feedback relationship between voltage and current [3]. Building on this foundation, we further integrate Markov processes and Monte Carlo methods to establish a load modeling framework capable of reflecting the stochastic nature of user behavior, while simultaneously introducing health state parameters to characterize the performance degradation process [4]. Through these methods, we develop a unified model system that combines physical interpretability with algorithmic scalability. We conduct analysis and validation using smartphone batteries as test cases, demonstrating the framework's applicability and stability under complex operating conditions.

2. Unified Algorithmic Framework for Electro-Thermal Battery Modeling

The battery discharge process of a smartphone is inherently a complex dynamic system involving electrochemical, thermal, and electrical interactions. Traditional Coulomb counting methods approximate the battery as an ideal capacitor, neglecting the nonlinear dependence of terminal voltage on load conditions. As a result, such approaches fail to explain phenomena such as rapid energy depletion under high load and unexpected shutdown under low-temperature conditions.

To address these limitations, a continuous-time electro-thermal coupled equivalent circuit model is established. The model captures three essential mechanisms:

Nonlinear voltage response: The terminal voltage depends not only on the state of charge (SOC), but also on ohmic voltage drop and polarization effects.

Constant power load behavior: The power management system operates approximately as a constant power load (CPL). As voltage decreases, current increases to maintain power demand, forming a positive feedback loop.

Electro-thermal coupling: Joule heating raises battery temperature, which in turn affects internal resistance and available capacity.

2.1. Electrical Dynamic Model

An enhanced first-order Thevenin equivalent circuit is adopted to characterize the battery dynamics. The model consists of an open-circuit voltage source, an ohmic resistance, and a polarization branch composed of a resistance-capacitance (RC) network [5].

(1) State of Charge Evolution

The SOC is defined as the ratio of remaining capacity to nominal capacity. Its evolution is governed by:

$$\frac{d\alpha(t)}{dt} = -\frac{I(t)}{3600 \cdot C_{\text{nom}}} \tag{1}$$

(2) Polarization Voltage Dynamics

The polarization voltage reflects electrochemical and concentration polarization effects:

$$\frac{dV_{\text{pol}}(t)}{dt} = \frac{I(t)}{C_{\text{pol}}} - \frac{V_{\text{pol}}(t)}{\tau} \tag{2}$$

(3) Terminal Voltage Equation

According to Kirchhoff's voltage law, the terminal voltage is given by:

$$V_{\text{out}}(t) = E_{\text{oc}}(t) - V_{\text{pol}}(t) - I(t)R_{\text{ohm}}(T,\alpha) \tag{3}$$

(4) Constant Power Load Constraint

Under the constant power assumption, the load current satisfies $I(t) = \frac{P_{\text{req}}(t)}{V_{\text{out}}(t)}$.

Substituting the voltage expression yields:

$$P_{\text{req}}(t) = I(t)\left[E_{\text{oc}}(t) - V_{\text{pol}}(t) - I(t)R_{\text{ohm}}(T)\right] \tag{4}$$

This formulation indicates that voltage reduction leads to current amplification, resulting in accelerated energy depletion.

2.2. Thermal Model

The battery temperature $\theta(t)$ is governed by the balance between internal heat generation and external heat dissipation:

$$MC_\theta \frac{d\theta(t)}{dt} = H_{\text{gen}}(t) - H_{\text{diss}}(t) \tag{5}$$

(1) Heat Generation

Joule heating is modeled as:

$$H_{\text{gen}}(t) = I(t)^2 R_{\text{eq}}(\theta(t)) \tag{6}$$

(2) Heat Dissipation

Convective heat transfer is expressed as:

$$H_{\text{diss}}(t) = \lambda S\left(\theta(t) - \theta_{\text{amb}}\right) \tag{7}$$

(3) Temperature-Dependent Resistance

The equivalent resistance follows a modified Arrhenius relationship:

$$R_{\text{eq}}(\theta) = R_0 \cdot \exp\left[\gamma\left(\frac{1}{\theta(t)} - \frac{1}{\theta_0}\right)\right] \tag{8}$$

The above equations establish the coupling between electrical and thermal dynamics, enabling accurate simulation of temperature evolution under varying operating conditions.

2.3. State-Space Representation

The system is formulated as a continuous-time state-space model with three state variables $X(t) = [\alpha(t), V_{\text{pol}}(t), \theta(t)]$.

(1) System Dynamics

The governing equations are:

$$\begin{cases} \dot{\alpha}(t) = -\frac{I(t)}{3600 \cdot C_{\text{nom}}} \\ \dot{V}_{\text{pol}}(t) = \frac{I(t)}{C_{\text{pol}}} - \frac{V_{\text{pol}}(t)}{\tau} \\ \dot{\theta}(t) = \frac{1}{MC_\theta}\left[I(t)^2 R_{\text{eq}}(\theta) - \lambda S(\theta(t) - \theta_{\text{amb}})\right] \end{cases} \tag{9}$$

(2) Current Solution under CPL

Substituting the voltage equation into the power constraint yields:

$$R_0 I(t)^2 + \left(V_{\text{pol}}(t) - E_{\text{oc}}(t)\right) I(t) - P_{\text{req}}(t) = 0 \tag{10}$$

The physically meaningful solution is:

$$I(t) = \frac{-\left(V_{\text{pol}}(t) - E_{\text{oc}}(t)\right) + \sqrt{\left(V_{\text{pol}}(t) - E_{\text{oc}}(t)\right)^2 + 4R_0 P_{\text{req}}(t)}}{2R_0} \tag{11}$$

(3) Termination Criterion

The time-to-empty (TTE) is defined as:

$$TTE = \min\{t \mid \alpha(t) \leqslant \alpha_{\text{cut}} \vee V_{\text{out}}(t) \leqslant V_{\text{cut}}\} \tag{12}$$

3. Stochastic Load Modeling and Markov-Modulated Monte Carlo Simulation Method

3.1. Markov-Modulated Monte Carlo Model

Based on the continuous-time electro-thermal model, the prediction of time-to-empty (TTE) under realistic usage scenarios is further investigated, with emphasis on stochastic load variation and battery aging effects.

User behavior introduces significant randomness due to frequent transitions among heterogeneous operating states, including standby, social interaction, media playback, and high-performance tasks. Deterministic load assumptions fail to capture this variability. Meanwhile, battery aging leads to capacity degradation and resistance growth, increasing the likelihood of voltage-triggered shutdown under high-load conditions.

To address these challenges, a Markov-modulated Monte Carlo framework is adopted. The operating states are represented as a finite state space: $S = s_1, s_2, \ldots, s_n$. Each state corresponds to a specific power profile. State transitions follow a continuous-time Markov process with transition probability matrix [6]:

$$P=\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix} \tag{13}$$

To capture intra-state variability, Gaussian noise is introduced:

$$P_{load}(t)=\bar{P}i(t)+\sigma_{\xi}(t)\xi(t),\quad \xi(t)\sim \mathrm{N}(0,1) \tag{14}$$

(1) Battery Aging Model

Battery health is characterized by $SOH\in(0,1]$. Its impact is modeled through two mechanisms:

Capacity degradation:

$$C_{eff}(SOH)=C_{nom}\cdot SOH \tag{15}$$

Resistance growth:

$$R_{age}(SOH,T,\alpha)=R_{nom}(T,\alpha)\cdot\exp\left(\kappa\left(\frac{1}{SOH}-1\right)\right) \tag{16}$$

(2) Termination Criterion

The TTE is defined as:

$$TTE=min\{t\mid \alpha(t)\le\alpha_{cut}\;;\vee;\;V_{out}(t)\le V_{cut}\} \tag{17}$$

This dual condition captures both energy depletion and voltage-limited shutdown, providing a realistic representation of device failure under high-load conditions.

3.2. Experimental Results

Monte Carlo simulations (N=50 per configuration) are conducted to evaluate the impact of key factors, including initial SOC, battery health (SOH), ambient temperature, and load intensity.

(1) Effect of Initial SOC

Table 1 summarizes the statistical results.

Table.1. Initial SOC results

Group	Mean	Variance	Minimum	Maximum
100% SOC	14.680283	2.5777708	8.4475	20.383333
50% SOC	6.071022	1.844019	2.3422222	10.8688889
80% SOC	10.793161	2.3002228	6.2025	14.983333

A strong positive correlation between initial SOC and TTE is observed. The average TTE decreases from 14.66 h to 10.79 h and further to 5.94 h as SOC decreases from 100% to 80% and 50%, respectively. This near-linear behavior is consistent with the relatively flat open-circuit voltage profile within the mid-SOC range.

The variance decreases with shorter TTE, indicating reduced accumulation of stochastic load fluctuations.

(2) Effect of Battery Aging (SOH)

As shown in Table 2, battery aging significantly degrades TTE beyond nominal capacity reduction.

Table.2. Battery health results

Group	Mean	Variance	Minimum	Maximum
Aged (70%)	9.6476661	1.897825	5.884167	14.4166667
New (100%)	15.215411	2.9766637	9.600833	20.8238889
Used (85%)	12.2196667	2.559616	6.9666667	19.321111

The average TTE decreases from 15.25 h (new battery) to 10.16 h (70% SOH), corresponding to a 33.4% reduction, exceeding the expected capacity loss. This behavior is primarily attributed to increased internal resistance, which accelerates voltage drop under high current conditions.

The distribution exhibits a pronounced lower tail, indicating an increased probability of premature shutdown under stochastic high-load conditions.

(3) Effect of Ambient Temperature

The results in Table 3 indicate a moderate temperature effect.

Table.3. Ambient temperature results

Group	Mean	Variance	Minimum	Maximum
Cold (0°C)	13.773556	2.442068	8.333333	17.611667
Hot (45°C)	15.439094	2.938161	9.333889	20.921389
Room (25°C)	14.976433	2.595860	10.367222	22.145000

At $0^{\circ}C$, the average TTE decreases to 13.64 h due to reduced electrochemical reaction rates and exponential growth of internal resistance. In contrast, the difference between $25^{\circ}C$ and $45^{\circ}C$ is negligible, suggesting stable performance within this temperature range.

(4) Effect of Load Intensity

As illustrated in Table 4, load intensity is the dominant factor influencing TTE.

Table.4. Load intensity results

Group	Mean	Variance	Minimum	Maximum
Gaming (1.5x)	8.839878	1.7625561	5.643333	12.5733056
Normal (1.0x)	14.378706	2.785387	9.3466944	20.550278
Power Save (0.7x)	21.225397	2.785387	18.106389	23.934722

The average TTE decreases from 14.38 h (normal load) to 9.39 h (gaming mode), corresponding to a 35.8% reduction. This effect results from both increased energy consumption and quadratic growth of Joule heating.

Additionally, higher variance under heavy load indicates amplified sensitivity to user behavior, leading to significant fluctuations in battery lifetime.

3.3. Model Validation and Discussion

To evaluate model accuracy, simulation results are compared with experimental discharge curves shown in Figure 1.

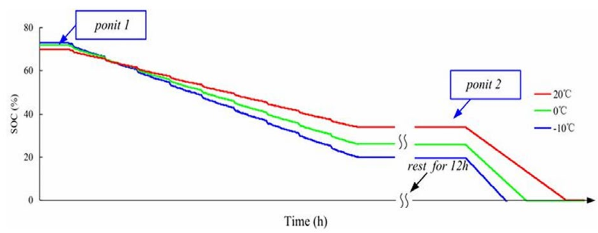

Figure 1. Discharge curves at different temperatures

(1) Validation against Experimental Data

The model predicts a 10.2% reduction in TTE when temperature decreases from $25^{\circ}C$ to $0^{\circ}C$, consistent with experimental observations. The reduced voltage plateau at low temperatures confirms the validity of the electro-thermal coupling mechanism.

(2) Nonlinear Behavior Analysis

When load intensity increases from 0.7× to 1.5×, TTE decreases from 16.75 h to 9.39 h, demonstrating strong nonlinear scaling. This is attributed to increased current, which amplifies both thermal losses and voltage drop.

Furthermore, the TTE at 50% SOC is significantly less than half of that at full charge, indicating nonlinear discharge behavior driven by increased current demand at low voltage levels.

(3) Model Limitations

The model does not explicitly capture transient voltage recovery under extreme low-temperature conditions (e.g., $-20^{\circ}C$), where self-heating may temporarily reduce internal resistance.

4. Conclusions

This paper proposes a unified modeling method for battery discharge that integrates electrical and thermal coupling mechanisms. Combined with a stochastic load modeling strategy, this approach enables a systematic characterization of battery behavior under complex operating conditions. First, by introducing an improved equivalent circuit and a temperature evolution model, the method captures the dynamic interrelationships among voltage, temperature, and internal resistance, effectively explaining performance variations under high-load and low-temperature conditions; Second, by incorporating constant-power load constraints, the study reveals the nonlinear feedback mechanism whereby voltage drop triggers current amplification, thereby modeling the accelerated decay phenomenon during discharge at the theoretical level; Subsequently, by constructing a Monte Carlo framework based on Markov process modulation and introducing stochastic perturbations and health state parameters, the model provides a unified description of load fluctuations and battery aging, ensuring good adaptability across various usage scenarios; Finally, simulation results indicate that as SOC decreases from 100% to 50%, the discharge duration exhibits a distinct nonlinear change, with the average duration decreasing by approximately 33% under aged conditions, thereby validating the model's effectiveness and stability in multi-factor coupled analysis. Future research may further investigate the impact of more complex environmental conditions and transient effects on the model.

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