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Adaptive Vibration Control of a Vertical Beam under Base Translation Excitation

Jiabao Wang[1, 2], Baoping Wang[1, 2]

[1]College of Engineering Machinery, Shandong Jiaotong University, Jinan 250357, China

[2]Shandong Provincial Engineering Research Center for Transportation Construction Equipment Technology and Intelligent Construction, Shandong Jiaotong University, Jinan 250357, China

Abstract

For a vibration control system of a vertical beam subjected to base excitation, a dynamic model incorporating the excitation term is established based on Euler–Bernoulli beam theory. Experimental results demonstrate the accuracy of the model. An adaptive PID control algorithm based on a Kalman filter is then proposed. The algorithm uses an inclination sensor to simultaneously acquire the beam's angular position and angular velocity. The Kalman filter provides optimal state estimates to mitigate the effects of measurement noise. Based on the filtered information, the vibration trend is predicted, and adaptive rules are constructed to adjust the PID parameters accordingly. Simulation and experimental results show that, compared with PID control, the proposed method significantly reduces the system settling time and effectively suppresses vibration.

Keywords

Vertical beam; Vibration control; Euler–Bernoulli beam; Kalman filter; PID control.

1. INTRODUCTION

Vibration control is one of the central technical challenges in modern mechanical engineering, particularly for upright beam systems widely used in engineering equipment. Upright beam structures are incorporated into a range of machinery, including forklift forks, loader booms, crane jibs, excavator implements, and robotic articulated arms. During load-bearing and operation, these structures inevitably experience vibration. Owing to their slender geometry, fixed support at the base, and compliant free ends, upright beam structures are susceptible to large-amplitude transverse vibrations under external excitation, abrupt start-up or shutdown, and payload variations. Such vibrations not only severely degrade operational accuracy and efficiency and compromise operator comfort, but may also induce structural fatigue damage, shorten equipment service life, and even result in safety incidents [1]. Previous studies have demonstrated that active control techniques can substantially improve the dynamic stability of such structures [2-4]. Therefore, active vibration control of upright beam structures is of considerable importance for enhancing the operational performance of engineering equipment, extending its service life, and ensuring operational safety.

Research on vibration control of beam structures has evolved progressively from passive control to active control and, more recently, to adaptive control. In passive control, several conventional devices, such as tuned mass dampers, viscoelastic dampers, and friction dampers, are commonly employed for vibration suppression [5-7]. Although these methods are structurally simple and reliable, they lack the adaptability required for time-varying systems [8, 9]. With the development of modern control theory, active control has attracted considerable

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attention as an effective approach, and numerous researchers have investigated the active vibration control of beam structures [10, 11]. Ma et al. [12] employed a cantilever beam as the research object and introduced a PID controller, experimentally verifying the effectiveness of closed-loop control in suppressing cantilever-beam vibrations. Fey et al. [13] investigated the vibration characteristics of a piecewise-linear beam system using a PD controller and demonstrated that active vibration control can reduce the adverse effects of vibration on the performance of beam systems. Huang et al. [14] employed an LQR controller to effectively suppress the vibration of a beam with partially covered active constrained-layer damping. However, fixed-parameter controllers have difficulty adapting to variations in system conditions and complex external disturbances. Improving the adaptability of the control system therefore requires the incorporation of adaptive control capabilities. Ali et al. [15] developed a fractional-order PID control method that effectively suppresses cantilever-beam vibrations, although the method entails a relatively high computational burden. Sangpet et al. [16] proposed an adaptive controller for the vibration control of piezoelectrically actuated flexible beams without relying on any approximate finite-dimensional model. Zhu et al. [17] addressed disturbances involving unknown multiple-frequency components by employing time-frequency analysis, and subsequently proposed an adaptive feedforward control scheme based on time-frequency analysis. In addition, the accuracy of system-state estimation is crucial to the overall control performance. As an optimal state-estimation algorithm, the Kalman filter can effectively mitigate the influence of measurement noise and has been widely applied in vibration-control systems. Chen et al. [18] incorporated a Kalman filter into a robust active vibration-control framework. Mohammadi et al. [19] proposed a Kalman-filter-based sensor-fusion method that integrates measurements from an accelerometer and an ultrasonic sensor, thereby improving the accuracy of shape estimation for a cantilever beam.

In summary, this study proposes a Kalman-filter-based adaptive PID control algorithm, hereafter referred to as the adaptive KFPID controller. The vertically oriented beam system subjected to base excitation is modeled as a rigid–flexible coupled system, and beam vibrations are suppressed by actively controlling the motion of the base. In contrast to existing approaches, a single-sensor configuration is adopted, in which an inclination sensor provides information on the beam angle and angular velocity, thereby simplifying the system architecture. Furthermore, a coordinated Kalman-filtering and adaptive-PID control strategy is developed. The Kalman filter is employed to obtain optimal estimates of the beam angle and angular velocity. These estimates are then used to anticipate the evolution of the vibration response and to formulate adaptive rules for the online adjustment of the PID parameters, thereby enhancing the system's disturbance-rejection capability.

2. DYNAMIC MODEL

An upright beam is a typical flexible structure, with one end fixed to a base and the other end free. When the base undergoes sudden stopping or variable speed, inertial forces are generated at the fixed end, causing the beam to undergo free vibration. Therefore, a dynamic model must be established to investigate the vibration control of this system. As shown in Figure 1. The beam dimensions considered in this study satisfy the slender-beam assumption. Moreover, under the assumption of small deformation, the effects of shear deformation and rotary inertia are neglected. Accordingly, the mathematical model is formulated based on Euler–Bernoulli beam theory.

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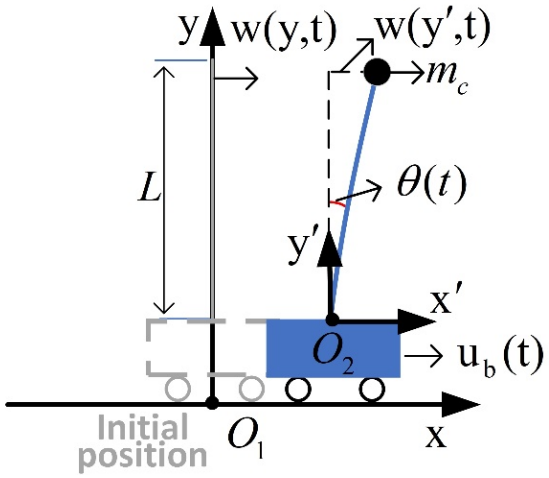


Figure 1. Schematic diagram of the vertical beam simulation model

In the inertial coordinate system, the absolute transverse displacement $w(y,t)$ can be decomposed into the rigid-body motion of the base and the bending deformation of the beam, as follows:

$$w(y,t) = u_b(t) + w(y',t) \tag{1}$$

where: $u_b(t)$ is the base displacement, and $w(y',t)$ is the deflection deformation of the beam relative to the base.

Based on the Euler–Bernoulli beam theory and D'Alembert's principle, the governing equation for forced vibration of a vertically oriented beam subject-ed to base translational excitation is derived:

$$\int_0^L \left[EI \frac{\partial^4 w(y',t)}{\partial y'^4} + \rho A \frac{\partial^2 w(y',t)}{\partial t^2} + \rho A \ddot{u}_b(t) \right] dy' + m_c \left[\frac{\partial^2 w(L,t)}{\partial t^2} + \ddot{u}_b(t) \right] = 0 \tag{2}$$

Where E is the elastic modulus, I is the moment of inertia of the cross-section, ρ is the material density, and A is the cross-sectional area. The base acceleration serves as external excitation input, inducing bending vibration in the upright beam through inertial coupling.

For a flexible, upright beam structure with a fixed base and a free top, the analytical expression of the i-th mode shape function that satisfies the associated boundary conditions is:

$$\phi_i(y') = \cosh(\beta_i y') - \cos(\beta_i y') - \sigma_i(\sinh(\beta_i y') - \sin(\beta_i y')) \tag{3}$$

where: $\sigma_i = \dfrac{\cosh(\beta_i L) + \cos(\beta_i L)}{\sinh(\beta_i L) + \sin(\beta_i L)}$, and β_i is the eigenvalue of the i-th order of the system.

The energy induced in a vertical beam system under dynamic excitation is predominantly concentrated within the low-frequency range; higher-order vibration modes contribute negligibly to the lateral displacement response and may therefore be omitted from analysis. Consequently, the relative lateral displacement can be approximated as:

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$$w(y',t) = \phi_1(y')q_1(t) \tag{4}$$

where: $q_1(t)$ is the first-order modal coordinate, and $\phi_1(y')$ is the first-order mode shape function.

Given that the upright beam structure possesses inherently weak damping characteristics, the small damping term is omitted in the modeling process. Substituting Equation (4) into Equation(2), and invoking modal orthogonality, the vibration control equation is derived.

$$m_1\ddot{q}_1(t) + k_1 q_1(t) = u_1(t) \tag{5}$$

where: $m_1 = \int_0^L \rho A\phi_1^2(x)dx + m_c\phi_1^2(L)$ is the first-order modal mass, m_c is the counterweight mass, and $u_1(t)$ is the generalized force vector generated by translational excitation, i.e., $u_1(t) = -\ddot{u}_b(t)[\int_0^L \rho A\phi_1(y')dy' + m_c\phi_1(L)]$.

For the subsequent vibration-controller design, a mathematical relation between the sensor-measured quantity and the transverse displacement of the beam is required. As illustrated in Figure 1, the sine of the positive tilt angle at the free end, $\sin\theta(t)$, is defined by the geometric ratio of the transverse displacement $w(L,t)$ to the beam length L . Under the small-deformation assumption, the trigonometric relation can be linearized—i.e., $\sin\theta(t) \approx \theta(t)$ — yielding the following expression [20]:

$$\theta(t) \approx \frac{w(L,t)}{L} = \frac{\phi_1(L)q_1(t)}{L} \tag{6}$$

To facilitate controller design, the system equations of motion are reformulated in state-space form. Defining the system state vector as $X = [q_1, \dot{q}_1]^T$, we obtain:

$$\begin{cases} \dot{X} = AX + Bf \\ Z = HX + v \end{cases} \tag{7}$$

where $Z = [\theta_s(t), \omega_s(t)]^T$ denotes the measurement vector, and $v = [v_\theta, v_\omega]^T$ denotes the measurement-noise vector. The matrices A , B , and H represent the state, input, and observation matrices, respectively, and are given explicitly as follows:

$$\begin{cases} A = \begin{bmatrix} 0 & 1 \\ -\dfrac{k_1}{m_1} & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 \\ \dfrac{1}{m_1} \end{bmatrix} \\ H = \begin{bmatrix} \dfrac{\phi_1(L)}{L} & 0 \\ 0 & \dfrac{\phi_1(L)}{L} \end{bmatrix} \end{cases}$$

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3. DESIGN OF ADAPTIVE KFPID CONTROLLER

3.1. Kalman filter

As a complex flexible structure, the vertically oriented beam system has unmeasured modal coordinates q_1 and $\dot{q}_1$ in its state vector X. In addition, measurement signals from the sensors are inevitably contaminated by noise. Directly using the raw measurements may therefore compromise the accuracy and stability of the control system. The Kalman filter provides an effective solution to this problem. By fusing the system's dynamic model with sensor measurements, it enables optimal estimation of the unmeasured system states. Moreover, its recursive formulation not only attenuates the effects of noise but also provides information regarding the reliability of the state estimates. Accordingly, a Kalman filter is incorporated to achieve optimal state estimation.

From the derived state space model, a discretized form is obtained:

$$\begin{cases} X_k = FX_{k-1} + Gf_{k-1} + w_{k-1} \\ Z_k = HX_k + v_k \end{cases} \tag{8}$$

where X_k denotes the state vector, F and G are the state-transition and control-input matrices, respectively, and f_{k-1} is the control input. The process noise w_{k-1} and measurement noise v_k are assumed to be zero-mean white Gaussian noise. Their covariance matrices are defined as $Q = diag(\overline{q}_1, \overline{q}_2)$, $R = diag(\sigma_\theta^2, \sigma_\omega^2)$. Here, $\overline{q}_1$ and $\overline{q}_2$ denote the process-noise variances associated with the modal coordinate q_1 and its time derivative $\dot{q}_1$, respectively, while σ_θ^2 and σ_ω^2 represent the measurement-noise variances of the angular displacement and angular velocity, respectively.

The Kalman filter obtains the optimal state estimate $\hat{X}_k$ through two steps: prediction and update. Because the actual sensor measurements are expressed in degrees, the estimated quantities are converted from radians to degrees. The resulting optimal estimates of the angle θ_f and angular velocity ω_f, which are used for control, are given by:

$$\begin{bmatrix} \theta_f \\ \omega_f \end{bmatrix} = H\hat{X}_k \cdot \frac{180}{\pi} \tag{9}$$

The detailed formula is unfolded as follows during the prediction phase:

$$\hat{X}_k^- = F\hat{X}_{k-1} + Gf_{k-1} + w_{k-1} \tag{10}$$

$$P_k^- = FP_{k-1}F^T + Q \tag{11}$$

Update phase:

$$K_k = \frac{P_k^- H^T}{HP_k^- H^T + R} \tag{12}$$

$$\hat{X}_k = \hat{X}_K^- + K_k(Z_k - H\hat{X}_K^-) \tag{13}$$

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$$P_k = (I - K_k H)P_k^- \tag{14}$$

where $\hat{X}_K^-$ denotes the prior state estimate, P_k^- is the prior error covariance matrix, K_k is the Kalman gain, $\hat{X}_k$ denotes the optimal posterior state estimate, and P_k is the posterior error covariance matrix.

3.2. Controller design

Because the PID controller employs fixed parameters, it cannot effectively respond to variations in system dynamics or external disturbances. Therefore, the PID control algorithm is combined with an adaptive control strategy to develop the adaptive KFPID controller. The controller takes the angle θ_f and angular velocity ω_f estimated by the Kalman filter as inputs, and dynamically adjusts the PID parameters in real time through the parameter tuner according to the adaptive law given in Eq.(15). The resulting control signal $u(t)$ drives the motion of the base and generates a force opposing the prevailing vibration tendency, thereby suppressing the vibration.

To enhance the predictive ability and adaptability of the controller, the following two auxiliary variables are constructed:

$$\begin{aligned} e_m &= |\theta_f| + \alpha |\omega_f| \\ \theta_p &= \theta_f + \omega_f \tau \end{aligned} \tag{15}$$

Adaptive PID control rules:

$$\begin{aligned} K_p &= K_{p0}(1 + a_p e_m) \\ K_d &= K_{d0}(1 + 0.5 e_m) \end{aligned} \tag{16}$$

Control output:

$$u(t) = K_p \theta_p + K_i \int \theta_f dt + K_d \omega_f \tag{17}$$

where e_m denotes the composite error used for adaptive parameter tuning, and θ_p is the predicted angular displacement at the future instant τ, obtained from a first-order Taylor expansion using the current angular velocity. The coefficient α is the weighting factor for the angular-velocity error, and τ is the prediction horizon. The parameters K_{p0}, K_i, and K_{d0} are the baseline gains determined through experimental tuning, while a_p denotes the adaptation rate used to regulate the gain adjustment. $u(t)$ is the output control.

4. SIMULATION AND EXPERIMENTAL VERIFICATION

4.1. Numerical simulation verification

The physical parameters of the vertically oriented beam system are listed in Table 1. The state-space model is truncated to the first vibration mode, and the governing equations of motion are numerically integrated using the classical fourth-order Runge–Kutta method.

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To validate the simulation model, a step force is employed to represent the base excitation. At the beginning of the simulation, an initial step input is applied, causing the beam to undergo forced vibration from its equilibrium position. As shown in Figure 2 and Figure 3, time-domain analyses are conducted on the angular displacement and angular velocity obtained from the simulations and experiments. The results demonstrate close agreement between the two datasets in terms of vibration frequency, amplitude attenuation, and phase characteristics. Furthermore, to further assess the accuracy with which the model captures the dynamic response and energy distribution of the system under low-frequency excitation, spectral analyses are performed on both the simulated and experimental data. As shown in Figure 5 and Figure 5, the dominant frequencies of the two datasets are nearly coincident, indicating a high degree of consistency. The comparisons in the time and frequency domains provide strong evidence for the accuracy and reliability of the proposed dynamic model.

This study compares the vibration-suppression performance of a conventional PID controller and the adaptive KFPID controller using the simulation model of the vertically oriented beam. The controller parameters are selected as follows. The parameters of the conventional PID controller are $K_p = 8500, K_d = 2200, K_i = 10$. The baseline parameters of the adaptive KFPID controller are $K_{p0} = 13000, K_{d0} = 3600, K_i = 50$, with an adaptation rate of $a_p = 0.2$, an angular-velocity weighting factor of $\alpha = 0.3$, and a prediction horizon of $\tau = 0.02$.

As shown in Figure 6 and Figure 7, the angular displacement and angular velocity responses under different control conditions are presented, respectively. The black curves represent the free-vibration responses of the system. The system exhibits an underdamped characteristic, resulting in a relatively slow decay of the vibration. The blue curves correspond to the vibration-control responses obtained with the conventional PID controller. Although the controller provides a certain degree of damping, the responses require approximately 2.5s to settle to their steady-state values. In comparison, the red curves represent the responses of the proposed adaptive KFPID controller and demonstrate a significant improvement. Throughout the control process, the peak responses are lower than those obtained with the conventional PID controller, and both the angular displacement and angular velocity settle to their steady-state values within approximately 1.5s. These simulation results indicate that, compared with conventional PID control, the proposed controller provides superior vibration suppression and improved dynamic response performance.

Table 1. Simulation data of inverted beam system

Parameter name	Parameter symbol	Numerical value
Concentrated mass	m_c	2.1kg
Beam length	L	700mm
Beam width	b	40mm
Beam thickness	h	3.7mm
Modulus of elasticity	E	2.10×1011Pa
Density	ρ	7850kg/m^3
Moment of inertia	I	168.9mm^4
Cross-sectional area	A	148mm^2

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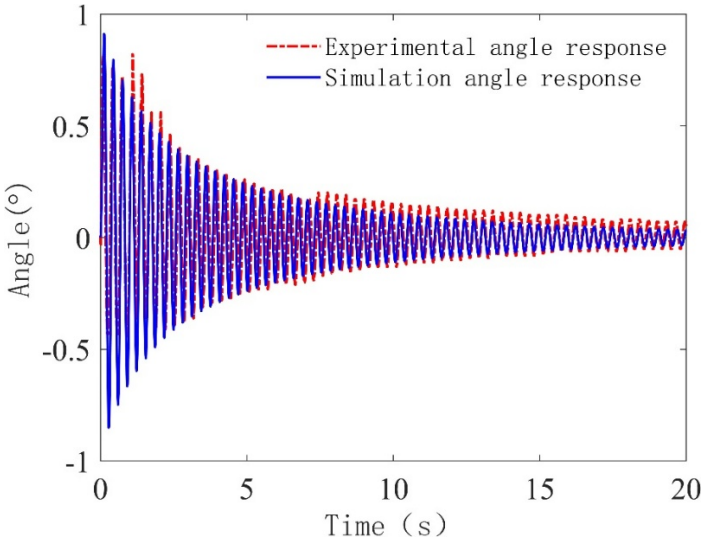


Figure 2. Time domain comparison of angle response

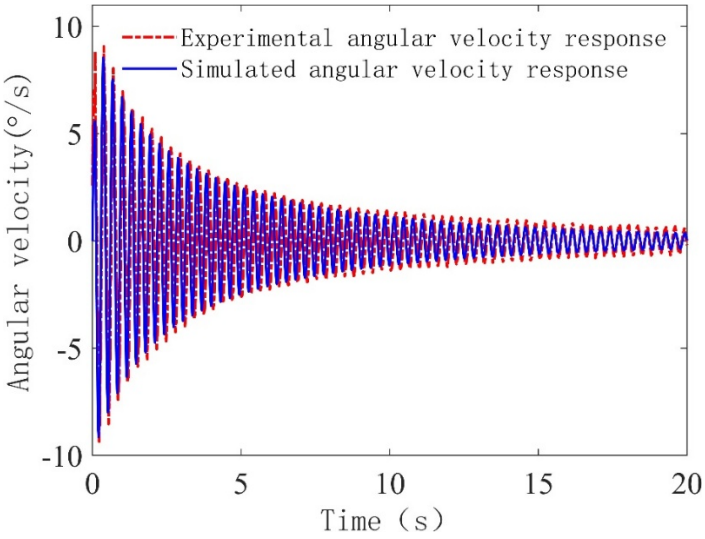


Figure 3. Time domain comparison of angular velocity response

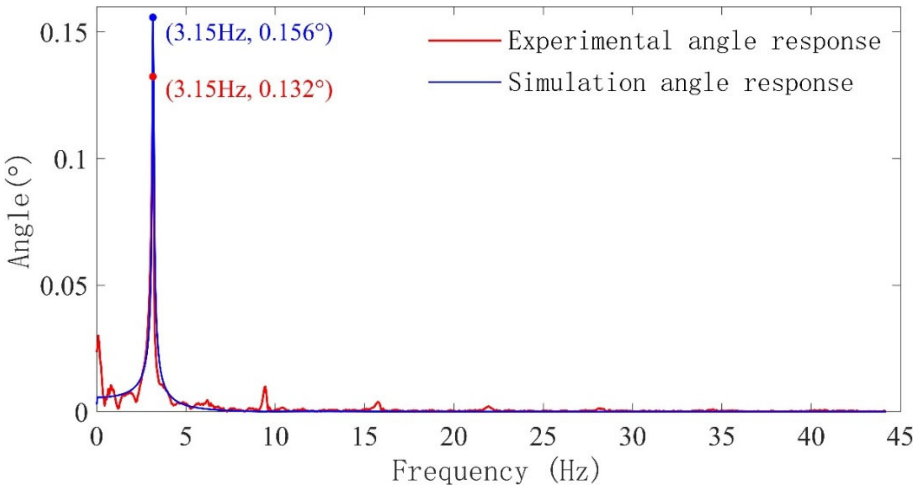


Figure 4. Frequency domain comparison of angular responses

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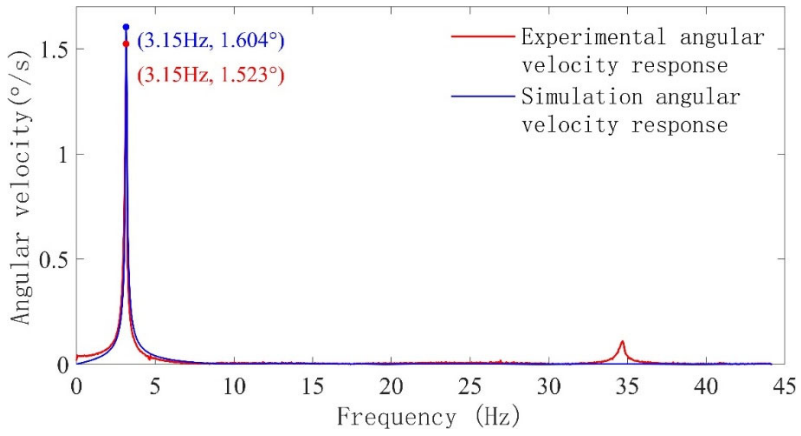


Figure 5. Frequency domain comparison of angular velocity response

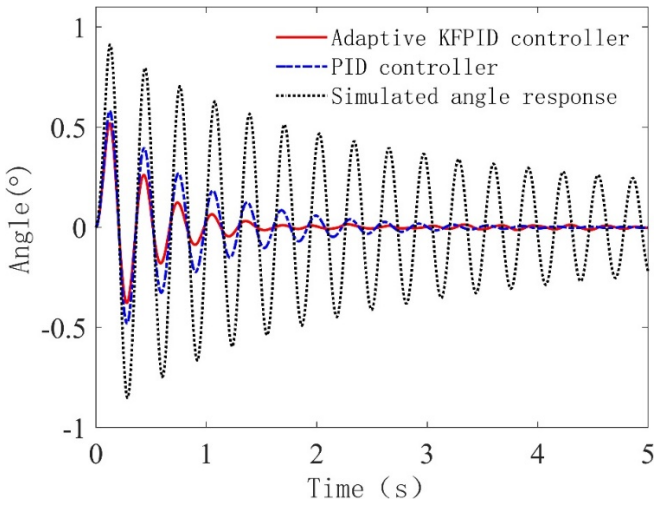


Figure 6. Comparison of Angle Control Effects of Different Controllers

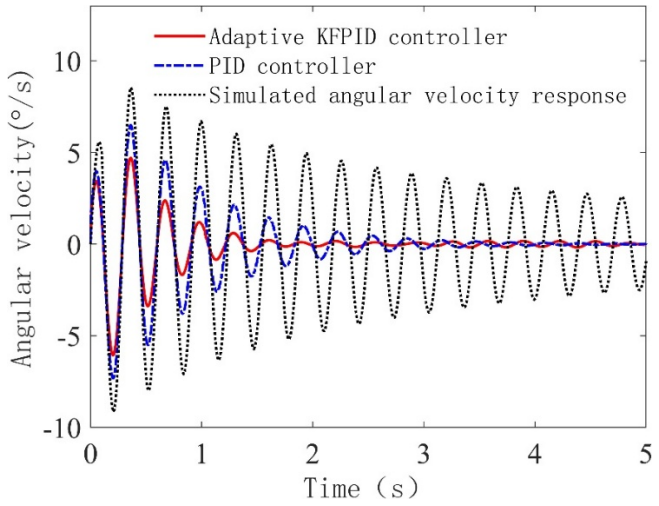


Figure 7. Comparison of angular velocity control effects of different controllers

4.2. Experimental verification

The experimental platform for the vertically oriented beam system is shown in Figure 8. The platform consists of a linear motor for driving the beam base and an inclination sensor for data acquisition. A communication link with the PC is established through a CAN communication

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module to facilitate data transmission and measurement recording. The bottom of the beam is rigidly connected to a PLC-controlled linear motor module, which communicates with the PC via the Siemens S7 protocol and performs two primary functions. First, when the motor is abruptly stopped, the resulting base excitation simulates an emergency-stop condition, thereby inducing transverse vibration in the beam. Second, the controller processes the acquired measurement data and generates the control command, which drives the base to move and produces an inertial force opposing the vibration, thereby rapidly suppressing the beam response. In the experiments, a PID controller and the proposed adaptive KFPID controller are implemented for comparative evaluation.

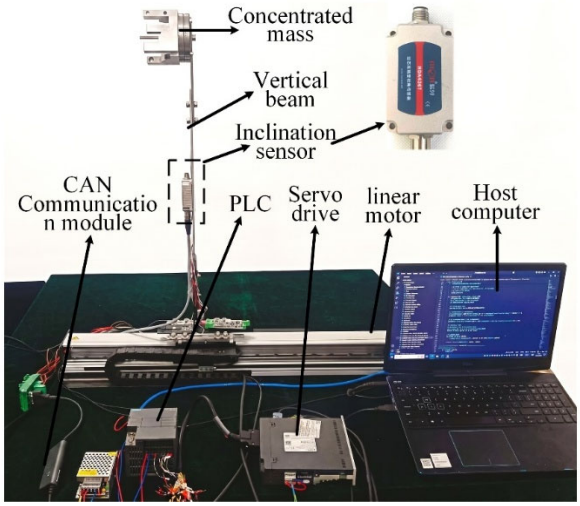


Figure 8. Vertical beam experimental platform

To evaluate the control performance of the proposed controller, comparative vibration-control experiments were conducted on the assembled vertically oriented beam test platform using the two controllers. Figure 9 and Figure 10 present the corresponding time-domain response curves of the angular displacement and angular velocity, respectively. The black curves represent the free-vibration responses of the uncontrolled system, the blue curves represent the responses obtained using the conventional PID controller, and the red curves represent those obtained using the proposed adaptive KFPID controller.

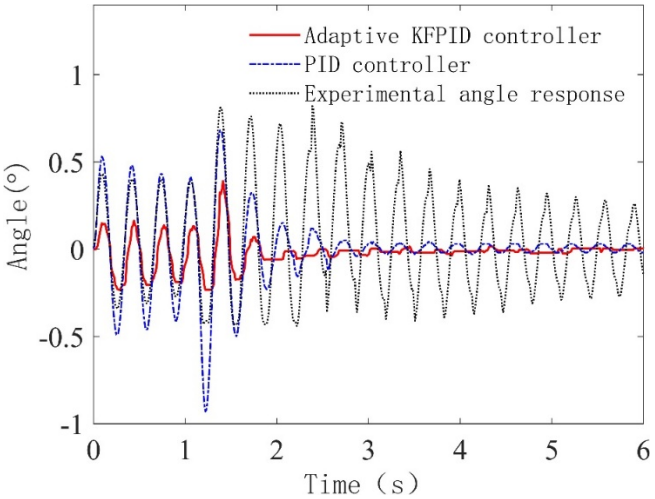


Figure 9. Vibration angle response curves of different controllers

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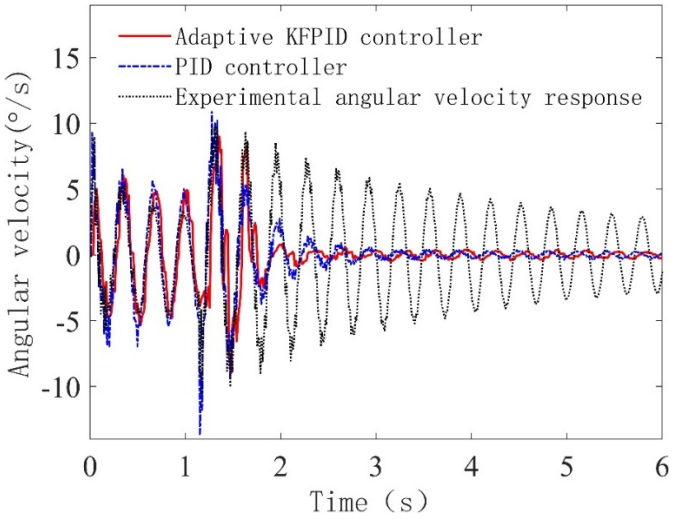


Figure 10. Vibration angular velocity response curves of different controllers

As shown in Figure 9, the angular displacement of the uncontrolled system exhibits sustained oscillations with a relatively slow decay rate. Under conventional PID control, the vibration is suppressed to some extent; however, the response follows the free-vibration trajectory slowly during the initial stage, and pronounced fluctuations occur at approximately 1.2s, with a peak amplitude of 0.9°. With the adaptive KFPID controller, the system rapidly tracks the desired motion trajectory at the initial stage. Throughout the control process, the angular displacement is effectively confined within 0.5°, resulting in a substantial reduction in vibration amplitude and an overall smooth decay response. Compared with PID control, the settling time is reduced by approximately 33%.

As shown in Figure 10, the angular-velocity response further confirms the superior performance of the proposed controller. In the absence of control, the vibrational energy associated with the angular velocity does not decay effectively. Under PID control, the vibration is reduced, but pronounced fluctuations occur at approximately 1.1s, with a peak angular velocity of 12°/s. This behavior indicates that the controller has limited capability to track changes in the system dynamics. In contrast, the proposed controller produces no severe fluctuations throughout the control process and yields a smooth decay response, with the system converging to within ±1°/s. The settling time is approximately 25% shorter than that achieved with PID control.

5. CONCLUSION

For the vibration control of a vertically oriented beam subjected to base excitation, an adaptive KFPID control algorithm was proposed and validated through numerical simulations and experiments. The main conclusions are as follows:

Based on Euler–Bernoulli beam theory, a dynamic model incorporating the base-excitation term was established and subsequently transformed into a state-space model, providing a theoretical foundation for controller design. The accuracy of the proposed model was verified through comparison with experimental results.

An adaptive KFPID control algorithm was developed. The algorithm uses sensor measurements of the beam angle and angular velocity to obtain optimal estimates of the system states through Kalman filtering. In addition, a trend-prediction-based adaptive mechanism was incorporated into the adaptive law, thereby enhancing the system's disturbance-rejection capability and adaptability.

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Experimental results demonstrate that, compared with the PID controller, the proposed controller reduces the settling times of the angular displacement and angular velocity responses by approximately 33% and 25%, respectively.

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